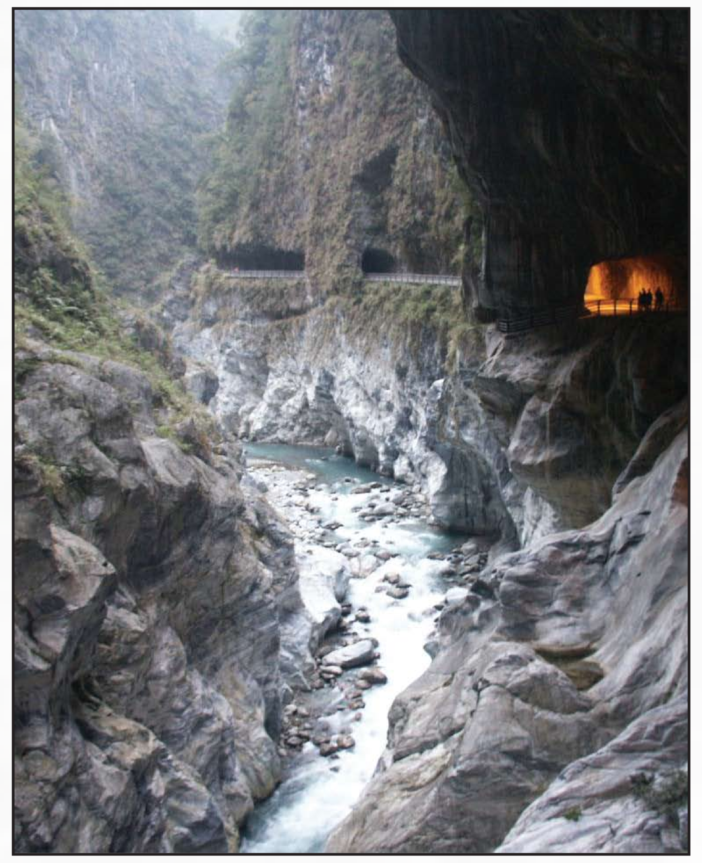
MODELING THE GEOMETRY OF BEDROCK RIVER CHANNELS

Cameron W. Wobus[1], Gregory E. Tucker[1,2], Robert S. Anderson[2,3]

[1]Cooperative Institute for Research in Environmental Sciences (CIRES), University of Colorado, Boulder
[2]Department of Geological Sciences, University of Colorado, Boulder
[3]Institute for Arctic and Alpine Research (INSTAAR), University of Colorado, Boulder

Abstract

We must understand the evolution of bedrock channel cross sections (or widths and shapes) in order to model properly the dependence of bedrock channel evolution on climate, tectonics, and sediment supply. We outline a modeling strategy to explore the origin of bedrock channel cross-sectional shapes, and to explain the dependence of channel width on flow discharge and local slope. Given a water discharge, bed material, and local slope, we calculate the mean velocity of the flow and then estimate the velocity gradients at all points along the wetted perimeter of the channel using the law of the wall. Assuming a shear-stress dependent erosion rule, this simplification of the fluid velocity structure allows us to calculate the spatial pattern of erosion rates along the wetted perimeter. The cross-sectional geometry and flow structure co-evolve until a steady cross-profile form is achieved. We then evaluate the dependence of these steady channel geometries on discharge, slope, and bed material properties. Our approach reproduces many of the scaling relationships observed in natural systems, including power-law width-discharge ($W \sim Q^{0.4}$) and width-slope ($W \sim S^{-0.2}$) relationships. For a fixed discharge, a greater slope requires a smaller cross section, and the flow narrows. The model also predicts a near-constant width-depth ratio for equilibrium channels. Models of channel cross-sections linked in series and subject to varying baselevel lowering rates, B, produce concave-up longitudinal profiles with power-law slope ($S \sim B^{1.31}$) and width ($W \sim B^{-0.24}$) dependence on rates of baselevel drop. We present premilinary results in which the model is extended to handle i) better representations of erosional processes, ii) better approximations of the flow structure, and iii) the role of non-uniform sediment mantling of the bed.



Liwu River, Taiwan

1. Modeling Approach

A. Calculate hydraulic radius and mean velocity of flow:

- Prescribe discharge (Q), slope (S), initial channel cross-section, and sediment size (D_{50})
- Use Chezy formulation to calculate hydraulic radius and mean velocity for all values of water depth (h) (Eqns. 1-2):

(1) $$\bar{u}_h = C_h \sqrt{R_h S}$$

(2) $$C_h = 5.75\sqrt{g}\,\log\left(\frac{12.2 R_h}{z_o}\right)$$

- Calculate the discharge for each value of water depth and minimize the misfit between prescribed and calculated Q (Eqn. 3):

(3) $$Q_h = \bar{u}_h A_h$$

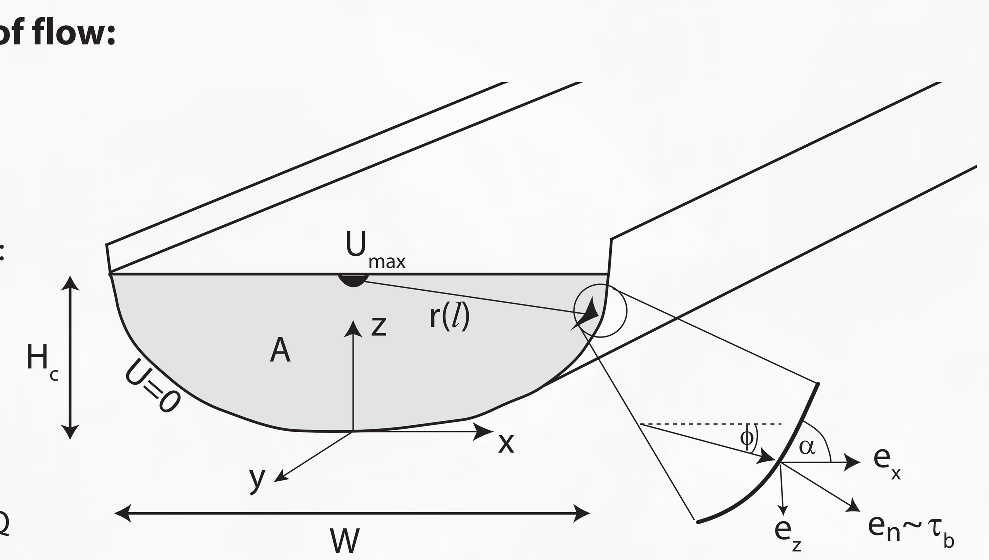


Figure 1: Model setup

B. Calculate shear stress and erosion rate distribution from velocity gradients near the wall:

- Use the law of the wall to calculate the maximum velocity from the mean velocity (Eqn. 4); then calculate the bed-normal component of the velocity gradient at a characteristic roughness height z_o (Eqn. 5):

(4) $$U_{\max} = \frac{\ln\left(h/z_o\right)}{\ln\left(h/z_o\right) - 1} \cdot \bar{u}$$

(5) $$\left.\frac{du}{dr(l)}\right|_{z_o} = \frac{u_*}{\kappa z_o} \cdot \sin(\phi - \alpha) = \frac{U_{\max}}{z_o} \cdot \frac{1}{\ln\left(r(l)/z_o\right)} \cdot \sin(\phi - \alpha)$$

- Following Prandtl, calculate the shear stress from the near-bed velocity gradient at each point along the bed (Eqn. 6); balance forces by ensuring that the integrated shear on the bed is matched by the downstream component of the weight of the water (Eqn. 7):

(6) $$\tau(l) = \varphi \rho A_h \left(\left.\frac{du}{dr(l)}\right|_{z_o}\right)^2$$

(7) $$\varphi = \frac{gS}{\sum_{i=1}^{N} \left(\left.\frac{du}{dr(l)}\right|_{z=z_o}\right)^2 dl(i)}$$

- Assume that the erosion rate scales linearly with the shear stress (e.g., Howard and Kerby, 1983)

2. Hydraulic scaling relationships

- Channel cross-sectional shape evolves to a smoothly concave-up form from any arbitrary geometry (Figure 2). Width-to-depth ratio is nearly constant for any prescribed value of grain size (Figure 3).

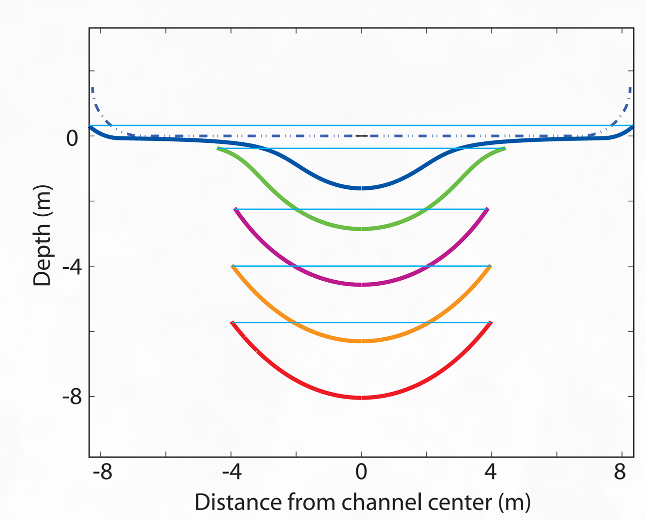


Figure 2: Attainment of steady-state geometry

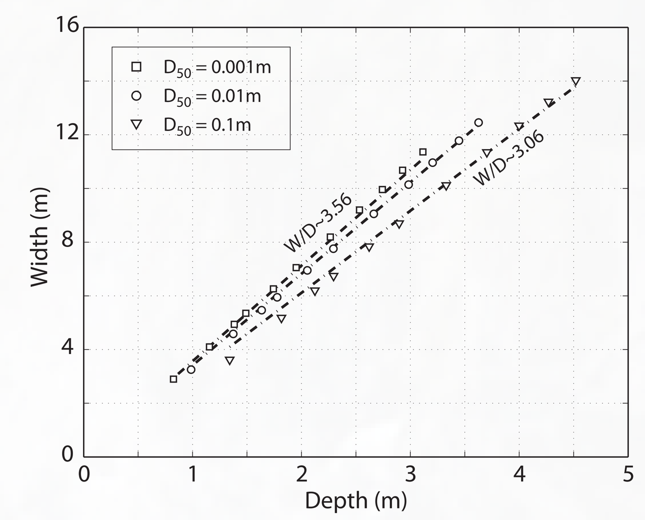


Figure 3: Nearly constant width-to-depth ratio

- Width vs. slope and width vs. discharge scaling relationships (Figures 4-5) are similar to those observed empirically, and are also similar to those predicted from analytical solutions that assume a constant width-to-depth ratio (e.g., Finnegan et al., 2005).

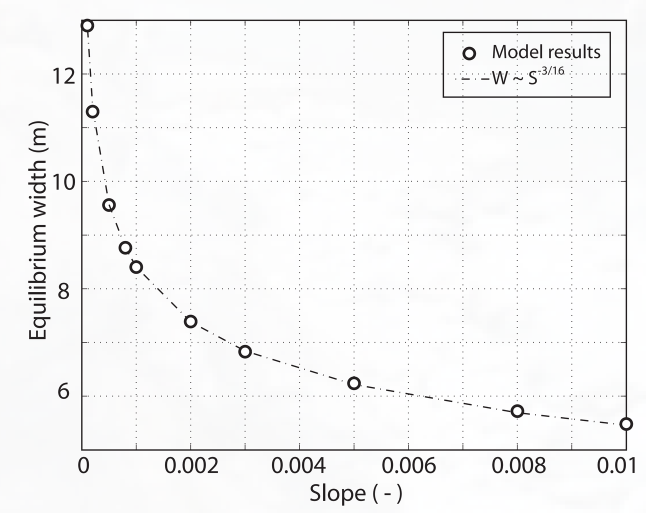


Figure 4: Width-slope scaling relationship

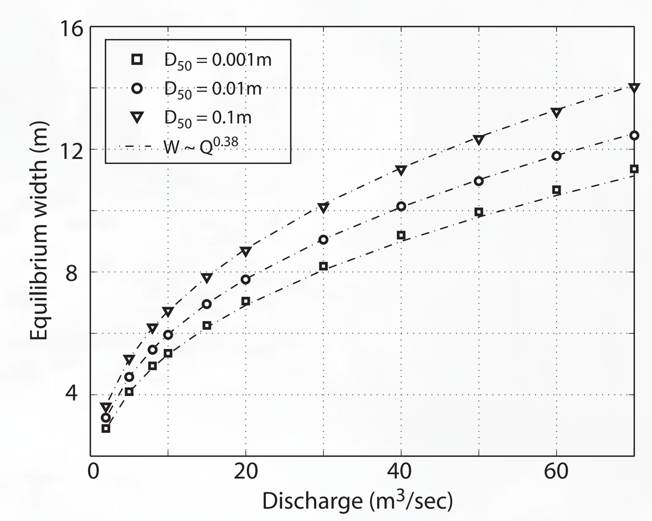


Figure 5: Width-discharge scaling relationship

3. Longitudinal profile evolution

Long profiles are modeled by linking 40 "V" shaped cross-sections along an initially linear ramp. Discharge increases downstream as $Q \sim y^{1.7}$, and the model is subjected to a spatially uniform rock uplift rate (or baselevel lowering rate), B. Width and gradient co-evolve in response to the imposed tectonic forcing.

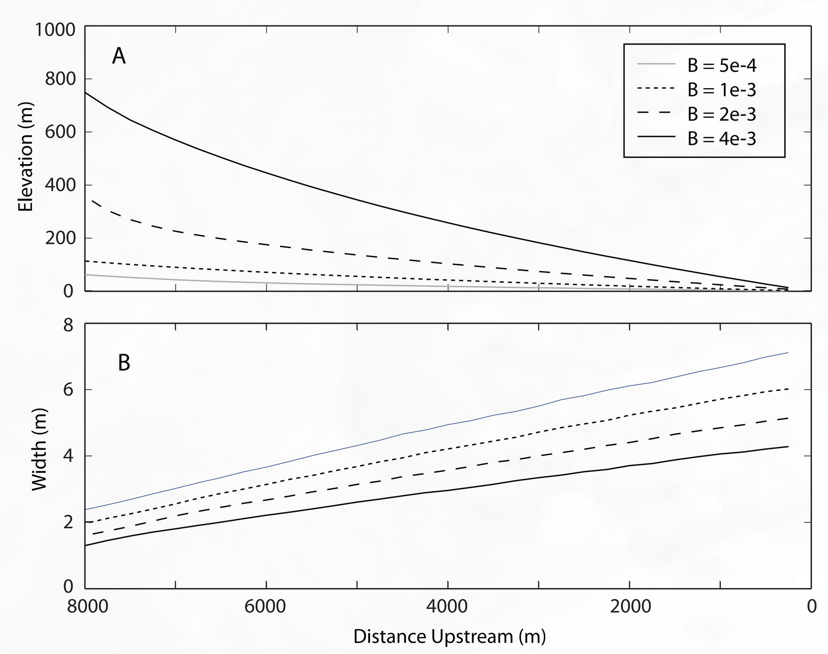


Figure 6: Steady-state longitudinal profiles (A) and width (B) under varying rock uplift rates

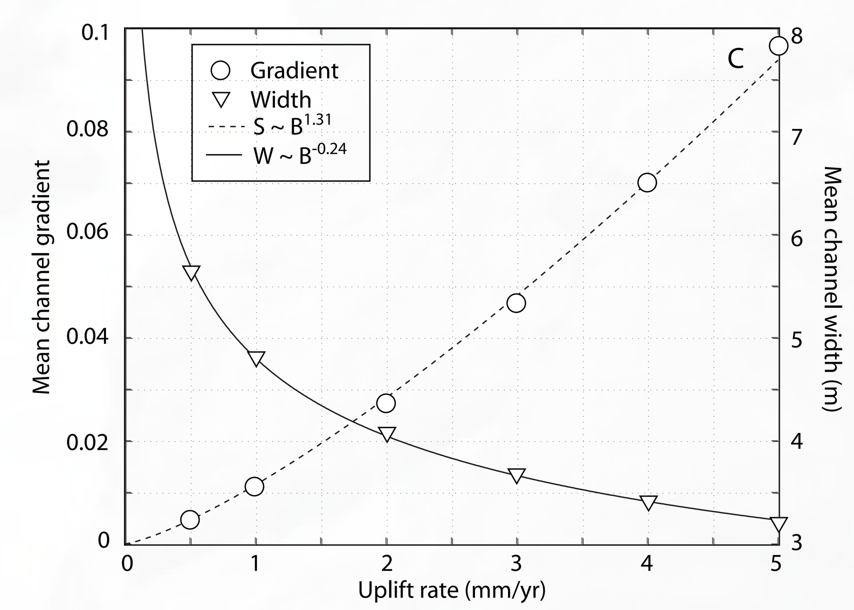
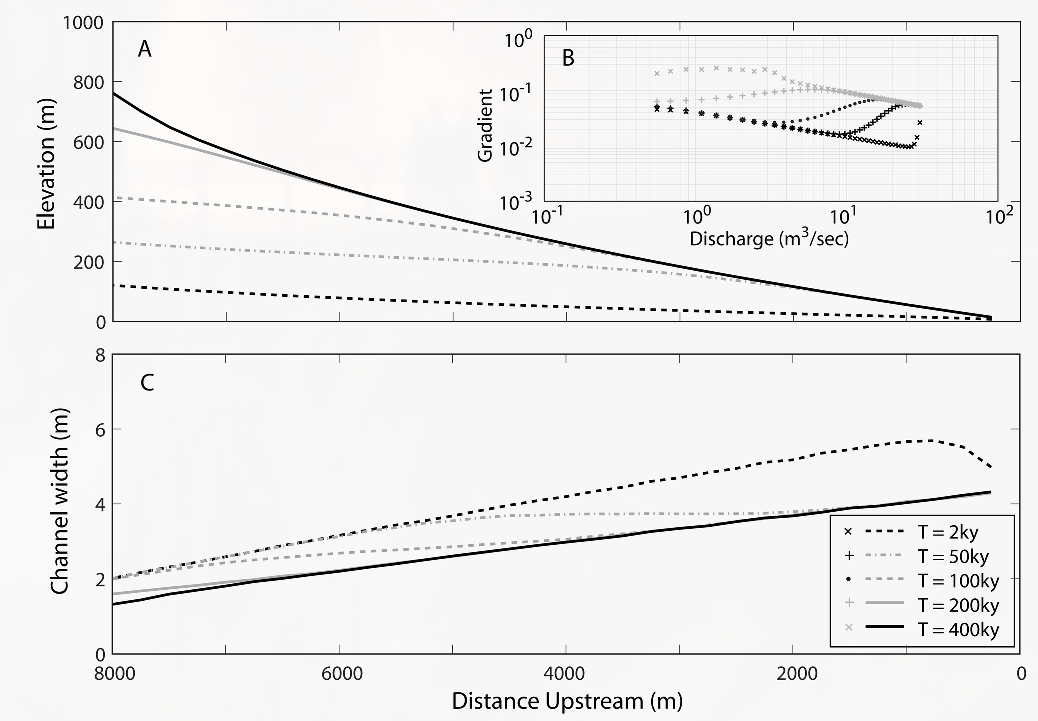


Figure 7: Width-uplift and Gradient-uplift scaling relationships at steady state

4. Transient response - Co-evolution of width and gradient



Model channels subjected to a change in rock uplift rate (or baselevel fall) respond by simultaneously narrowing and steepening as an upstream-migrating wave of incision (Figures 8A and 8C). Log-log plots of slope vs. discharge evolve in a similar manner to slope-area relationships in theoretical studies (e.g., Whipple and Tucker, 1999) and empirical studies (e.g., Wobus et al., 2006) of transient channel response (Figure 8B).

Figure 8 - Transient response to a change in rock uplift rate (relative to baselevel)

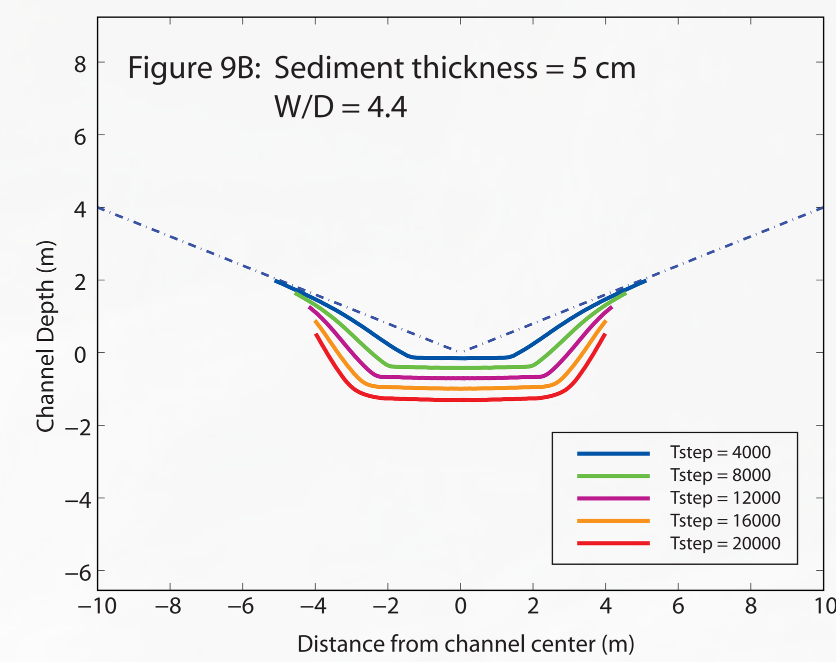
5. Effects of sediment cover on steady-state cross-sectional form

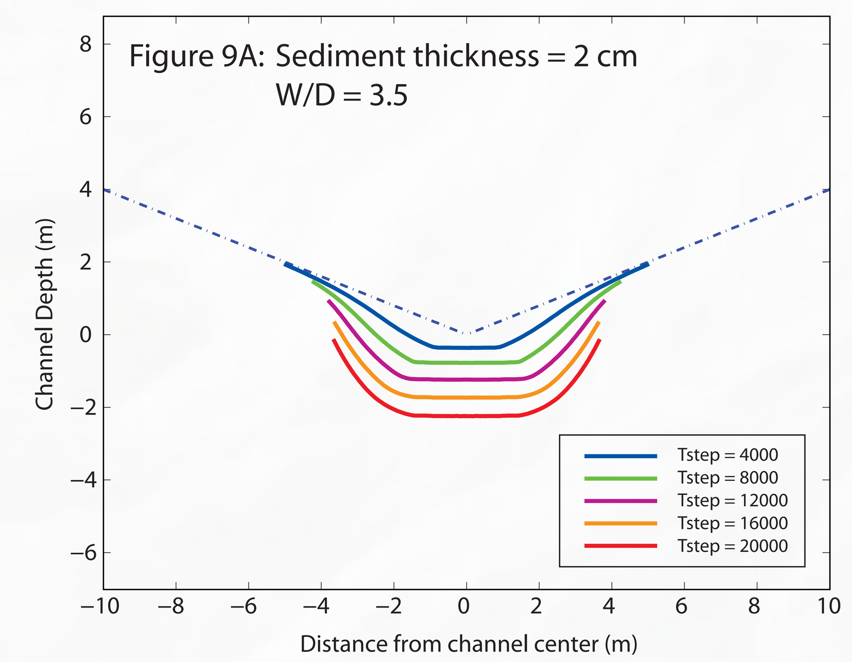
We have begun to model how sediment on the bed influences steady-state hydraulic geometry. We employ an empirical observation that the distribution of scour depths in gravel-bed rivers follows an exponential distribution, whose model parameter is related to the reach-averaged shear stress (e.g., Haschenburger et al., 1999):

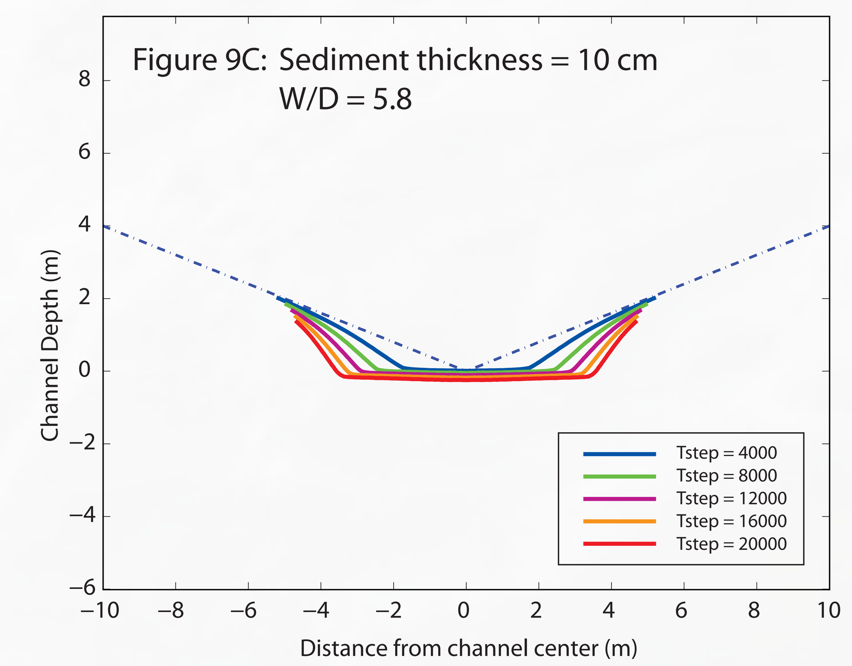
(8) $$f(x) = \theta e^{-\theta x}$$

(9) $$\theta = A e^{-B\tau^*/\tau_r^*}$$

We prescribe a sediment thickness on the channel bed. At each timestep, the scour depth at each node is then picked from a random distribution that is weighted according to Equations 8-9. The bed is available for attack only if the scour depth exceeds the sediment thickness. Figures 9A-C illustrate the effect of this parameterization on hydraulic geometry: holding all else constant, a thicker sediment mantle causes model channels to shallow and widen, and decreases the lowering rate of the channel.







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Haschenburger, J. K. (1999), A probability model of scour and fill depths in gravel-bed channels, Water Resources Research, 35, 2857-2869.
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Support for this work came from a CIRES postdoctoral fellowship to CWW, NSF grants #EAR-0510971 (GT) and EAR-0545537 (RSA) and USARO grant #47033-EV (GT)