THE BEST CHOICE OF THE SPLITTING PARAMETER IN THE EWALD METHOD

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Abstract Submission Form

2004 National Radio Science Meeting

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Date Received: September 23, 2004

The Ewald method is a powerful means to efficiently evaluate the free-space periodic Green's function (FSPGF). In the Ewald method, the FSPGF is expressed as the sum of a "modified spectral" and a "modified spatial" series. Each series possesses Gaussian exponential decay and this leads to an overall series representation that exhibits very rapid convergence. The overall convergence is optimum when the asymptotic rate of convergence of each series is the same, and this occurs when using the "optimum" value of the Ewald spitting parameter E given by $E_{\text{opt}} = \sqrt{\pi/A}$, where A is the cross-sectional area of the periodic lattice cell.

However, one problem with the Ewald method is that at high frequency (when the period becomes large relative to a wavelength) the numerical accuracy degrades quickly. This is due to a loss of significant figures in the summation of the two series, since the two series converge to very large and nearly opposite values. This, in turn, is caused by the $(0,0)$ terms in each of the two series becoming very large. This fundamental observation was first noticed in (A. Kustepeli and A. Q. Martin, *IEEE Microwave and Guided Wave Letters*, **10**, 168-170, May 2000).

A method is proposed here for choosing the "best" value of E (denoted here as E_L) that gives the fastest convergence of the Ewald sum, while limiting the number of significant figures L that are lost to a specified level (e.g., $L = 3$). In particular, the method determines the value of E_L that is required to limit the size of the $(0,0)$ terms in each of the two series relative to the numerical value of the total Greens function, in order to limit the loss of significant figures to the specified level.

In order to limit the size of the $(0,0)$ term in the spatial series of the Ewald method, the parameter E should be larger than a particular value called E_{spat}, which can be approximated in closed form. Similarly, in order to limit the size of the $(0,0)$ term in the spectral series, the parameter E should be larger than a particular value called E_{spect}, which can also be approximated in closed form. The best value for the parameter E is then given by the formula

$$E_L = \max\left(E_{\text{opt}}, E_{\text{spect}}, E_{\text{spat}}\right).$$

The loss of significant digits has been verified through numerical simulations for both planar and non-planar cases, and the results verify the accuracy of the above formula.

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2. B - Fields and Waves

3. (a)

4. C - Contributed Paper

5. No special instructions