

# Atmospheric models

Chem 5151 - Christoph Knote - NCAR  
Spring 2013

based on:

Introduction to Atmospheric Chemistry, D. Jacob, Princeton Press, 1999

Prof. Colette Heald's course <http://www.atmos.colostate.edu/~heald/teaching.html>



NCAR

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total change in  
X over time

## Continuity equation

$$\frac{dX}{dt} = F_X + E_X + P_X - L_X - D_X$$

**Nothing is ever lost.**

The change of a quantity  $X$  over time  
is the net result of *sources minus sinks*.

Source and sink terms depend on the application.

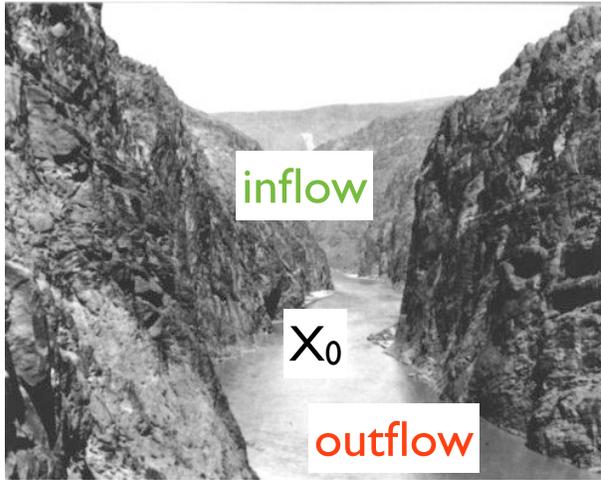
total change in  
X over time

# Transport

$$\frac{dX}{dt} = F_X + E_X + P_X - L_X - D_X$$

**transport**

Example (X == water)



$F_{in}$  (inflow) -  $F_{out}$  (outflow)

*in this example  $\approx 0.0$*

<http://www.usbr.gov/lc/hoverdam/images/p45-300-02371.jpg>

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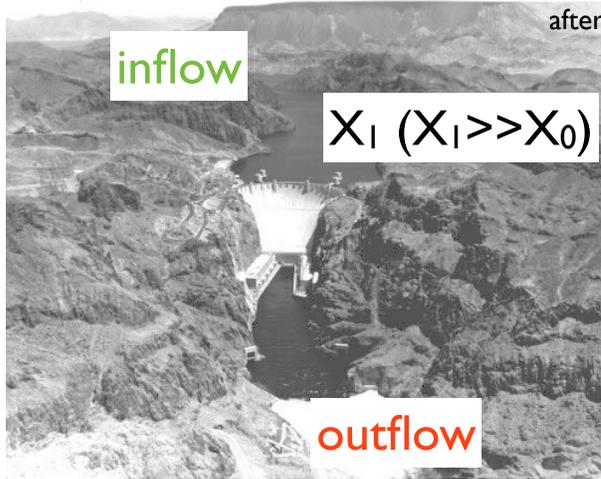
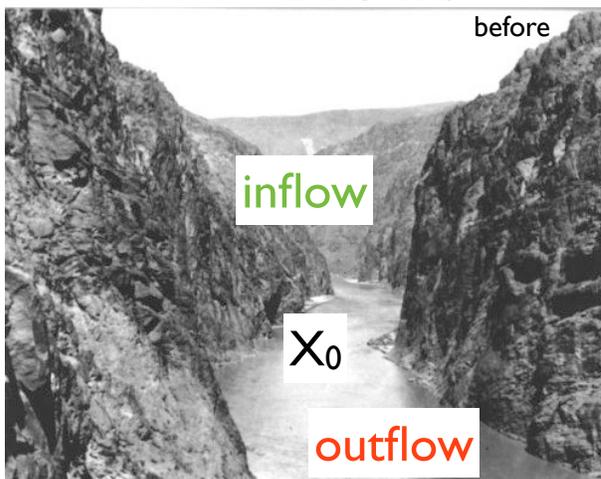
total change in  
X over time

# Transport

$$\frac{dX}{dt} = F_X + E_X + P_X - L_X - D_X$$

**transport**

Example (X == water) + Hoover dam



<http://www.usbr.gov/lc/hoverdam/images/p45-300-02371.jpg>

<http://www.usbr.gov/lc/hoverdam/images/p45-300-01501.jpg>

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## Clicker question

Did the transport term  $F_X$  (on average) change after construction of the Hoover dam?

- A. yes, it is now  $> 0.0$
- B. yes, it is now  $< 0.0$
- C. no
- D. yes, it is now  $= 0.0$
- E. I don't know

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- C. no
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- E. I don't know

**C no (on average),  
because the reservoir would  
overflow if the term is still  
positive once the reservoir is  
full.**

**transport is always:  
 $F_{in} - F_{out}$**

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total change in  
X over time

# Emissions

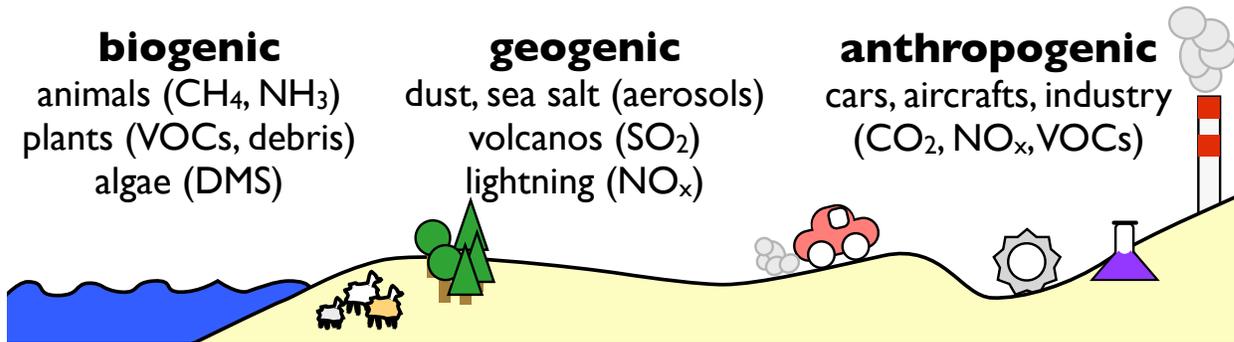
$$\frac{dX}{dt} = F_X + E_X + P_X - L_X - D_X$$

**emissions**

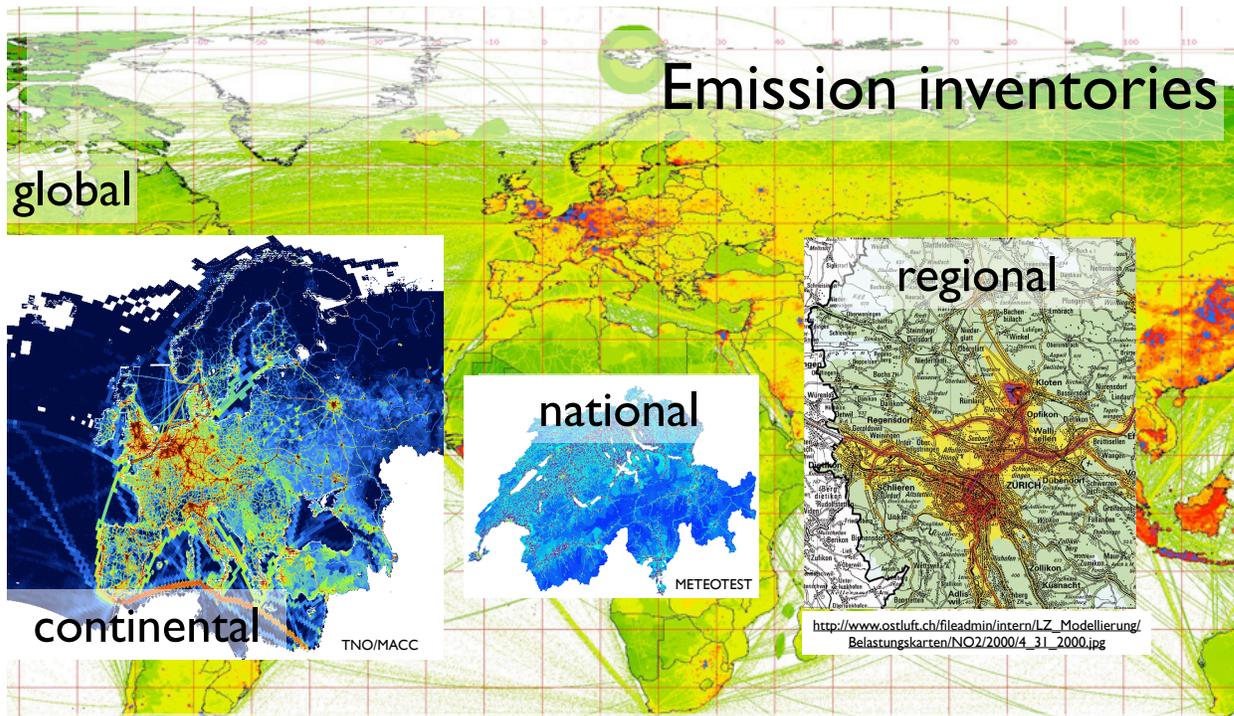
based on **emission factors** and **activity rates**:

$$E_X = EF_X \cdot A_X = 368 \text{ kg CO}_2 \text{ emitted}$$

Example: 368 g CO<sub>2</sub> / km      1.000 km driven



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different datasets on different grids for certain species,  
multiple (overlapping) datasets for a single species

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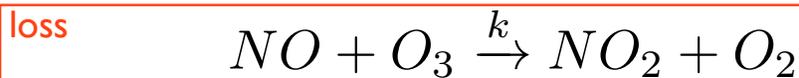
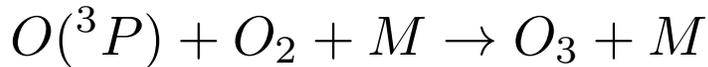
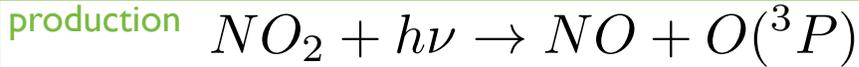
total change in  
X over time

# Chemical reactions

$$\frac{dX}{dt} = F_X + E_X + P_X - L_X - D_X$$

**chemical Production / Loss**

e.g. tropospheric ozone (photostationary state)



in our model e.g.:

$$= J_{NO_2}[NO_2] + f([O(^3P)], [O_2], [M]) - k_{NO,O_3}[NO][O_3]$$

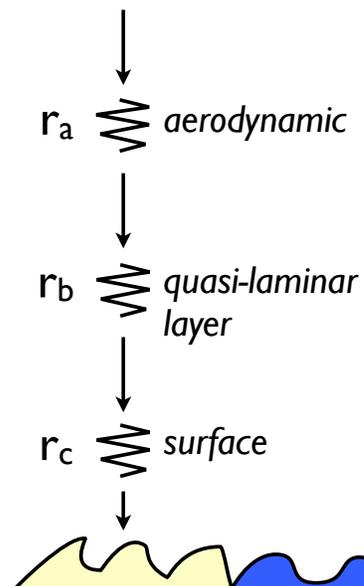
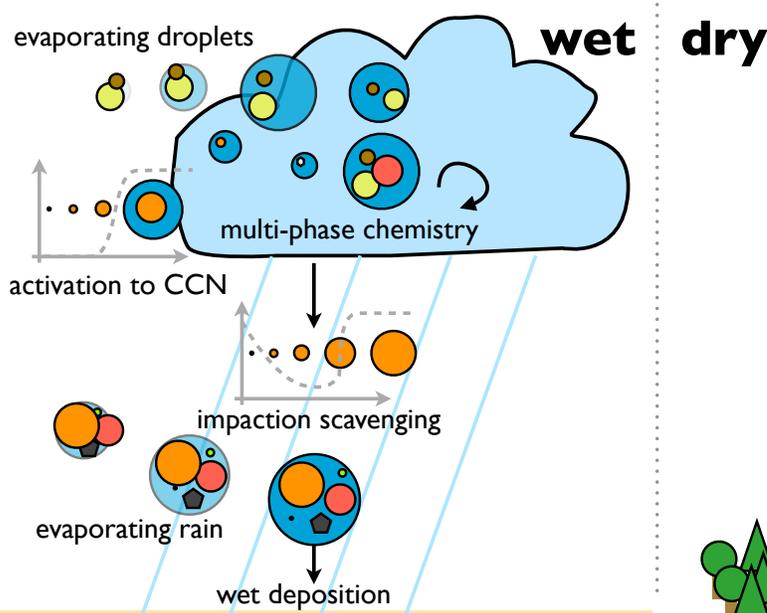
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total change in  
X over time

# Deposition

$$\frac{dX}{dt} = F_X + E_X + P_X - L_X - D_X$$

**deposition**



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# Resistance approach

Calculation of dry dep. flux  $F_X = v_{dep} [X]$   
 deposition velocity      concentration of X

based on resistance analogy  $r_{total} = v_{dep}^{-1}$

$$F_X = \frac{X_3 - X_2}{r_a} = \frac{X_2 - X_1}{r_b} = \frac{X_1 - X_0}{r_c} = \frac{X_3 - X_0}{r_{tot}}$$

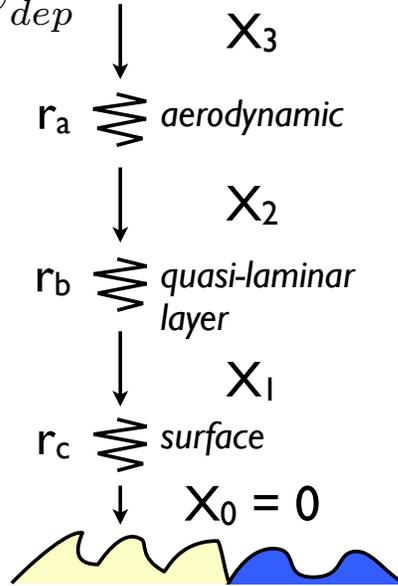
$$= \frac{X_3 - (F_X r_b + X_1)}{r_a}$$

$$= \frac{X_3 - (F_X r_b + F_X r_c)}{r_a}$$

$$F_X = \frac{X}{r_a + r_b + r_c}$$

resistances in series

$$v_{dep} = \frac{1}{r_a + r_b + r_c}$$

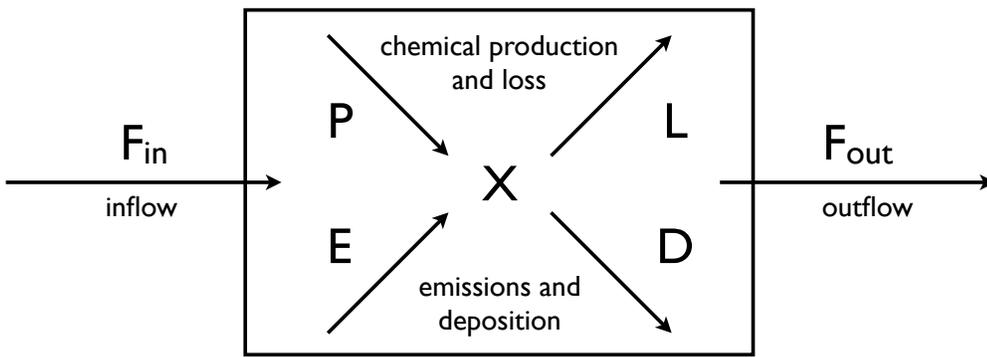


typical values of  $v_{dep}$  for gases: 0.5 cm/s

see also: Wesely and Hicks, Atm. Env., 34, 2000



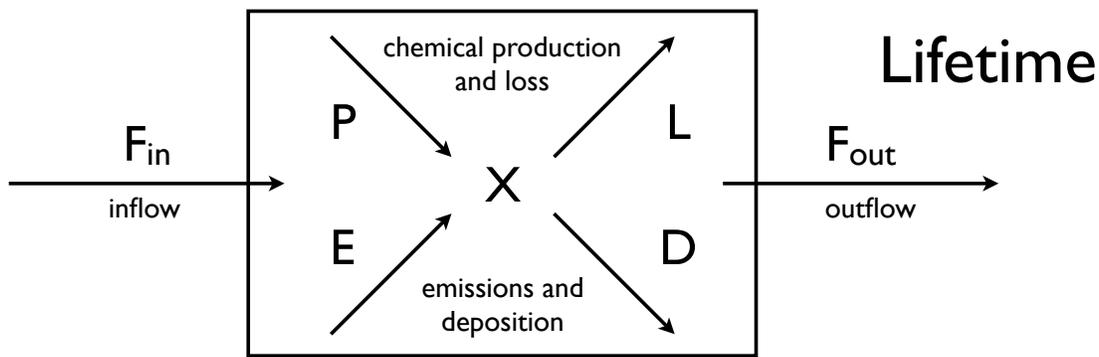
# Box models



adapted from Introduction to Atmospheric Chemistry, D. Jacob

All variables (e.g. X) and environmental conditions (temperature, pressure, ...) are homogeneous within the box:

**„well-mixed“ assumption**



adapted from Introduction to Atmospheric Chemistry, D. Jacob

mass balance:

$$\frac{dX}{dt} = \sum \text{sources} - \sum \text{sinks} = F_{in} - F_{out} + E_X + P_X - L_X - D_X$$

$$\text{atmospheric lifetime: } \tau = \frac{X}{F_{out} + L_X + D_X} \text{ (s)}$$

$$\text{lifetimes add in parallel: } \frac{1}{\tau} = \frac{1}{\tau_{outflow}} + \frac{1}{\tau_{chemical loss}} + \frac{1}{\tau_{deposition}}$$

$$\text{loss rates relate to lifetime: } k = \frac{1}{\tau} \text{ (s}^{-1}\text{)}$$

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## Clicker question

Box of 1 x 1 x 1 m. No transport.

**20 ppbv** of compound X in the box at steady state.

Chemical production of X: **0.1 ppbv/s**. Chemical loss (first order k): **0.005 1/s**. Emissions of **0.5 ppbv/s**.

Deposition: **0.025 ppbv/s**.

Lifetime of X against chemical loss?

- A. 4000 s
- B. 200 s
- C. 160 s
- D. 12000 s
- E. I don't know

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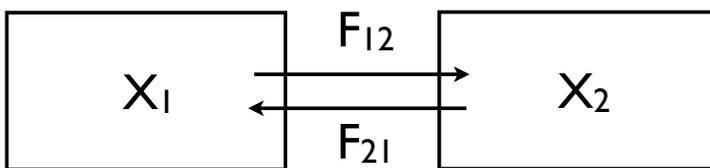
$$\frac{1}{\tau} = \frac{1}{\tau_{outflow}} + \frac{1}{\tau_{chemical\ loss}} + \frac{1}{\tau_{deposition}}$$

$$\tau_{chemical\ loss} = \tau_{L_x} = \frac{X}{L_X}$$

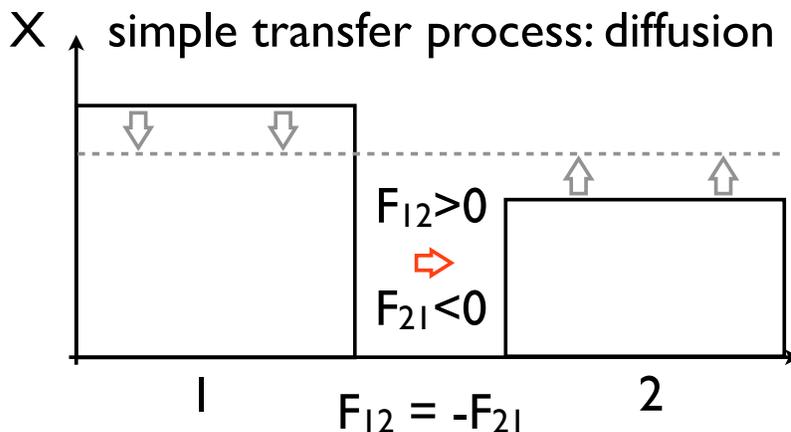
**B**

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## Two-box models

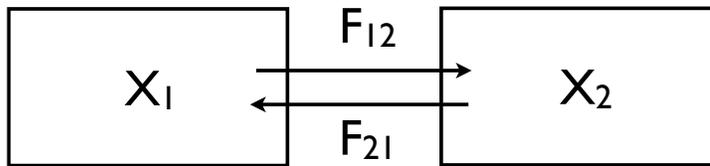


adapted from Introduction to Atmospheric Chemistry, D. Jacob



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# Two-box models



adapted from Introduction to Atmospheric Chemistry, D. Jacob

extended continuity eq. for  $X_1$

$$\frac{dX_1}{dt} = E_1 + P_1 - L_1 - D_1 + F_{21} - F_{12}$$

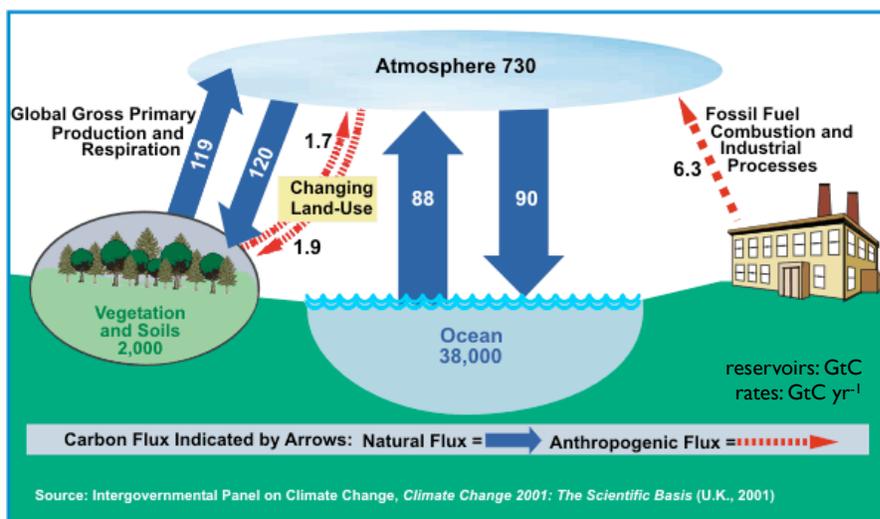
assume flux is mass times transport coefficient (k)

$$\frac{dX_1}{dt} = E_1 + P_1 - L_1 - D_1 + k_{21}X_2 - k_{12}X_1$$

**system of coupled ODEs**  
(analytically solvable if in steady state)

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## Examples of box models



adapted from <http://www.esrl.noaa.gov/research/themes/carbon/>

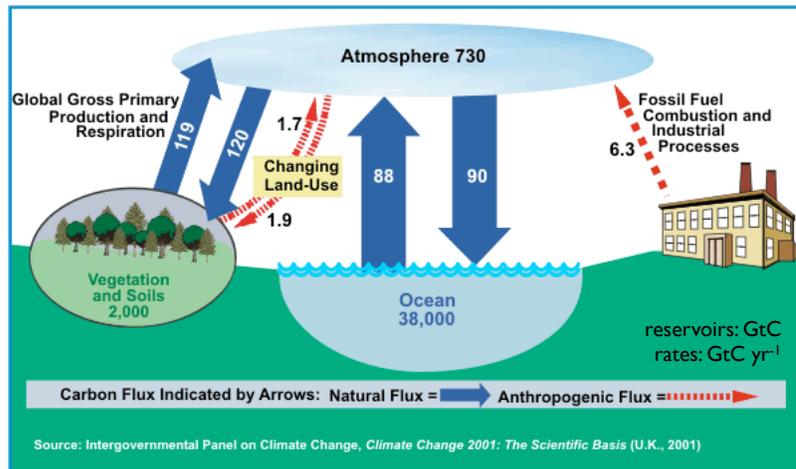
- hemispheric exchange studies, emission estimates
- lifetime and lifecycle assessments for longer lived species (e.g. CO<sub>2</sub>, HFCs, CH<sub>4</sub>)

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# Clicker question

What is the lifetime of C in the atmosphere?

- A. 1.6 yrs
- B. 3.4 yrs
- C. 2.8 yrs
- D. 4.9 yrs
- E. I don't know



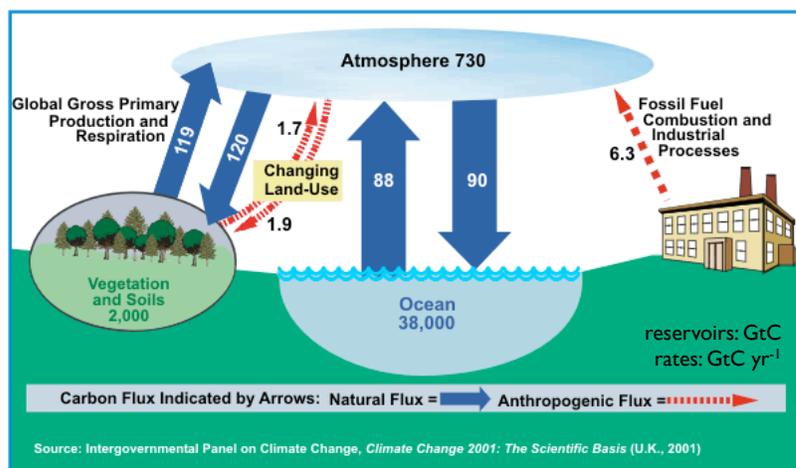
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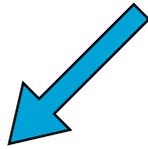
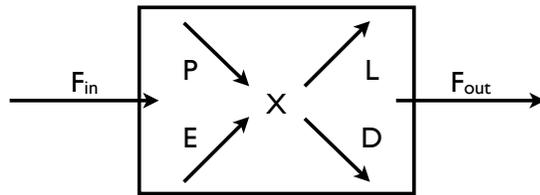


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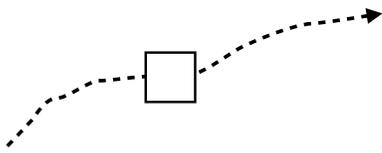
B. 
$$= \frac{730}{90 + 120 + 1.9} = 3.45 \text{ yrs}$$

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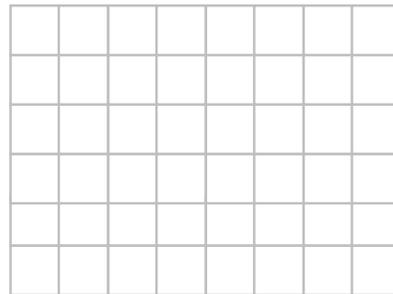
## box models



## Lagrangian models

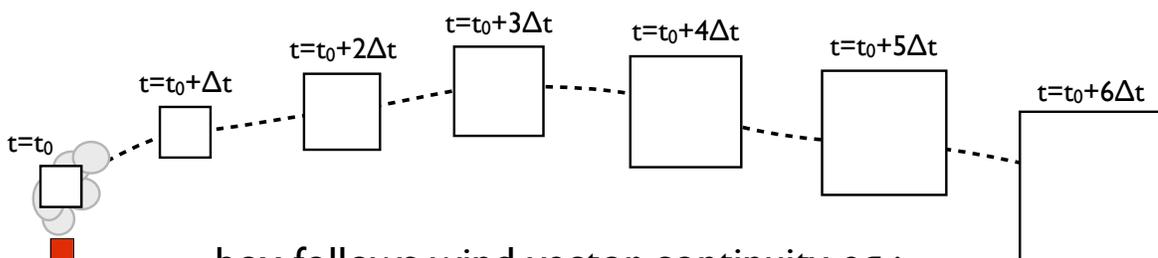


## Eulerian models



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## Puff models



box follows wind vector, continuity eq.:

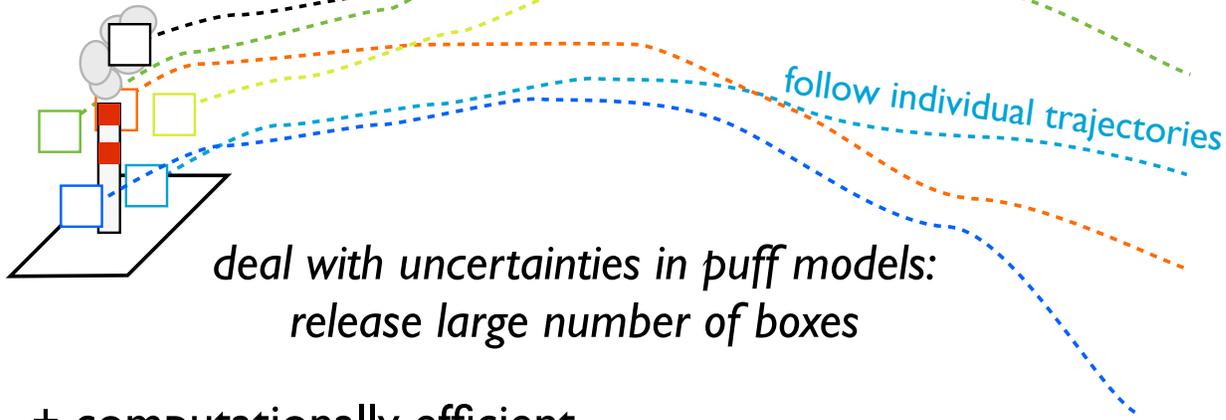
$$\frac{dX}{dt} = E + P - L - D - k_{dil}(X - X_{env})$$

**dilution** term - air in box mixes with surroundings

Large uncertainties in transport and dilution possible!  
(additionally to uncert. in chemistry, emissions, removal)

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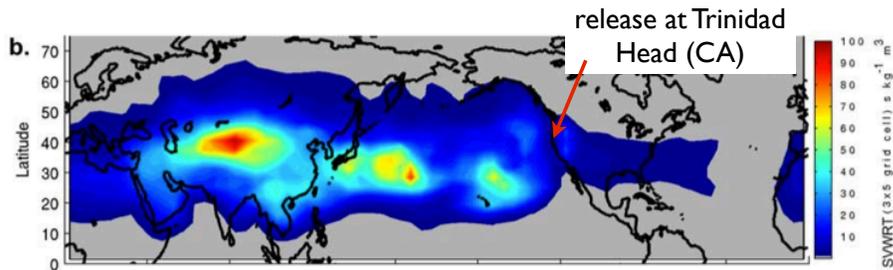
# Lagrangian models



*deal with uncertainties in puff models:  
release large number of boxes*

- + computationally efficient
- no mixing between individual boxes (chemistry!)
- statistical undersampling away from source

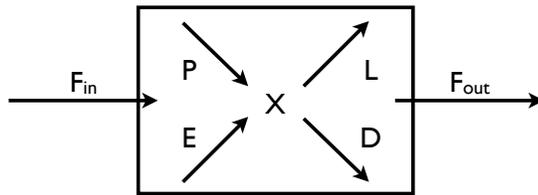
23



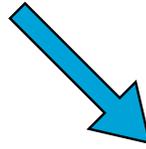
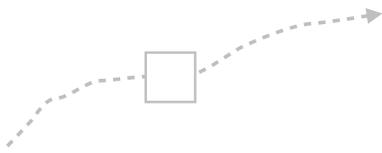
**Example**  
residence time

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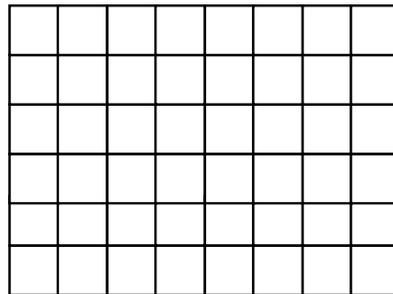
## box models



## Lagrangian models

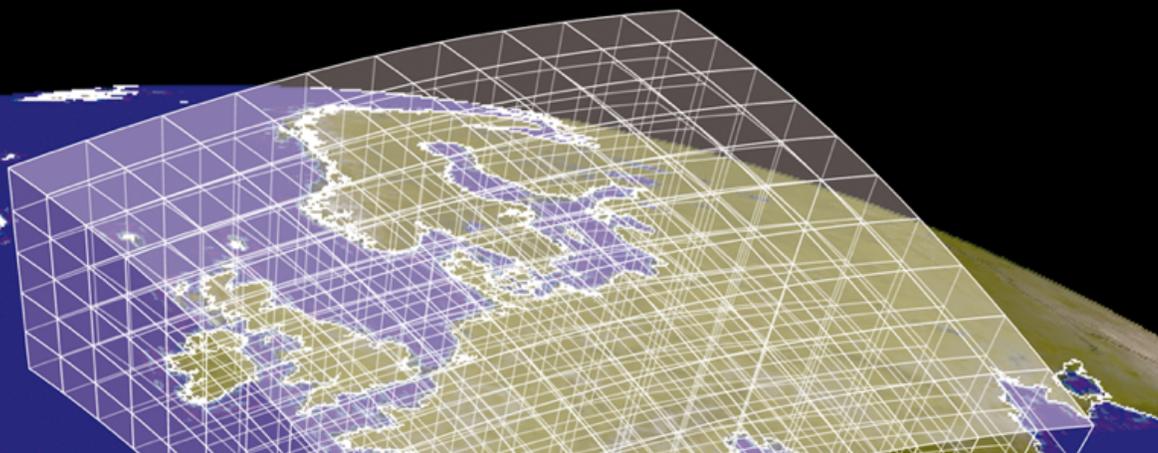


## Eulerian models



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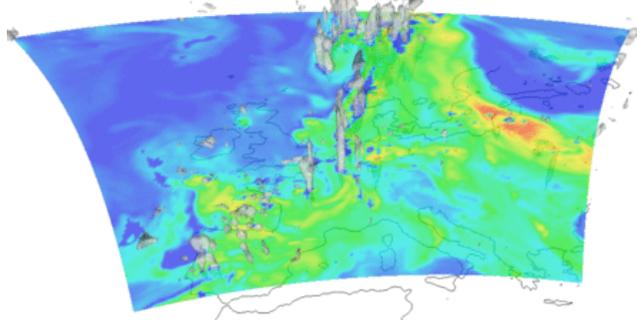
## Eulerian 3D models



system of coupled ODEs (for each box)  
~ $10^6$  boxes are computationally feasible currently  
typical box sizes: 20-100 km (global), 1-100 km (regional)  
vertical extent: surface to 10 - 0.01 hPa

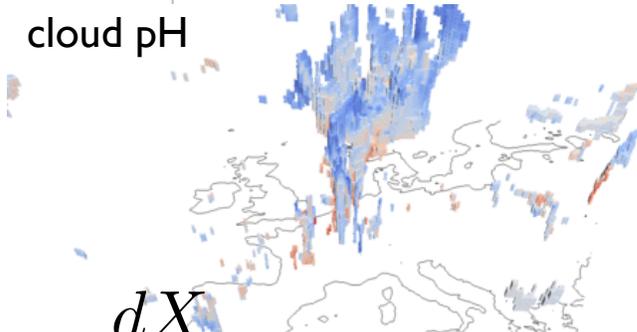
SO<sub>4</sub><sup>2-</sup> (z ≈ 1 km)

# Eulerian model example



processes seen here:  
 transport, convection ( $F_X$ )  
 wet deposition ( $D_X$ )  
 aq.-phase chem. ( $P_X$ )

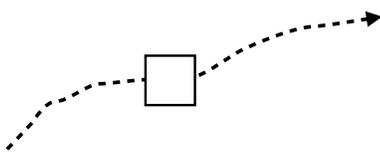
cloud pH



$$\frac{dX}{dt} = F_X + E_X + P_X - L_X - D_X$$

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## Lagrangian models

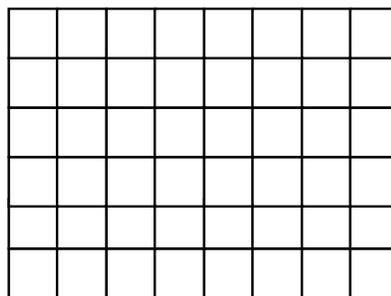


! output of Eulerian model („weather forecast“) necessary for wind vector !

boxes follow wind vector,  
 transport term == 0

- + computationally efficient
- + no numerical diffusion
- no mixing (chemistry)
- statistical undersampling

## Eulerian models



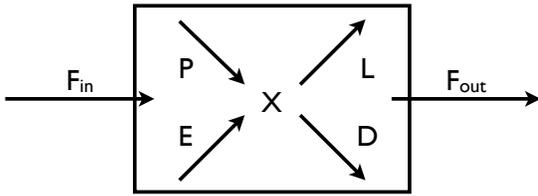
large # of boxes at fixed positions

- + complex chemistry, interactions possible
- expensive computation
- numerical diffusion

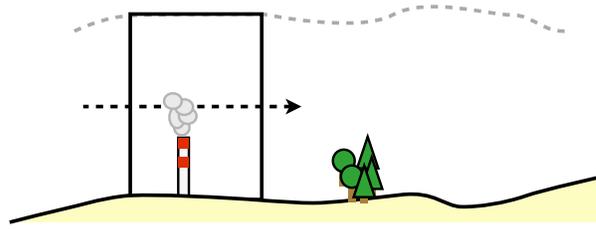
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# Final overview

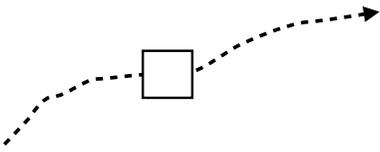
## Box models



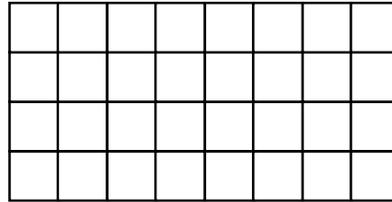
## Column models



## Lagrangian models



## Eulerian models



various types of models, you decide depending on question you want to answer