

7 Properties of the Atmospheric Aerosol

7.1 THE SIZE DISTRIBUTION FUNCTION

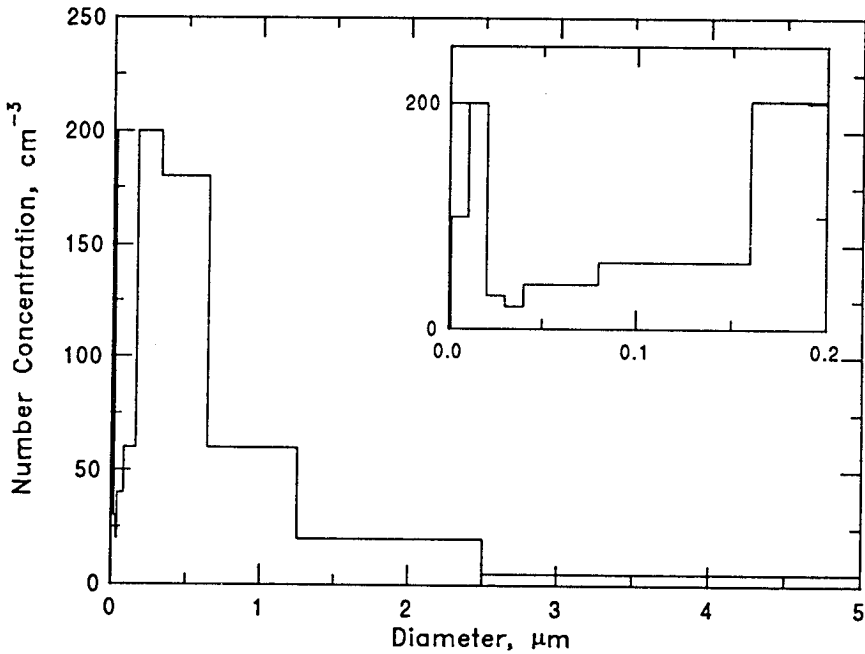
The atmosphere, whether in urban or remote areas, contains significant concentrations of aerosol particles sometimes as high as 10^7 to 10^8 cm^{-3} . The diameters of these particles span over four orders of magnitude, from a few nanometers to around $100 \mu\text{m}$. To appreciate this wide size range one just needs to consider that the mass of a $10 \mu\text{m}$ diameter particle is equivalent to the mass of one billion 10 nm particles. Combustion-generated particles, such as those from automobiles, power generation, and wood burning, can be as small as a few nanometers and as large as $1 \mu\text{m}$. Wind-blown dust, pollens, plant fragments, and sea salt are generally larger than $1 \mu\text{m}$. Material produced in the atmosphere by photochemical processes is mainly found in particles smaller than $1 \mu\text{m}$. The size of these particles affects both their lifetime in the atmosphere and their physical and chemical properties. It is therefore necessary to develop methods of mathematically characterizing aerosol size distributions. For the purposes of this chapter we neglect the effect of particle shape and consider only spherical particles.

An aerosol particle can be considered to consist of an integer number k of molecules or monomers. The smallest aerosol particle could be defined in principle as that containing two molecules. The aerosol distribution could then be characterized by the number concentration of each cluster, that is, by N_k , the concentration (per cm^3 of air) of particles containing k molecules. Although rigorously correct, this discrete method of characterizing the aerosol distribution cannot be used in practice because of the large number of molecules that comprise even the smallest aerosol particles. For example, a particle with a diameter of $0.01 \mu\text{m}$ contains approximately 10^4 molecules, and one with a diameter of $1 \mu\text{m}$ around 10^{10} .

A complete description of the aerosol size distribution can also include an accounting of the size of each particle. Even if such information were available, a list of the diameters of thousands of particles, that would vary as a function of time and space, would be cumbersome. A first step in simplifying the necessary accounting is division of the particle size range into discrete intervals and calculation of the number of particles in each size bin. Information for an aerosol size distribution using 12 size intervals is shown in Table 7.1. Such a summary of the aerosol size distribution requires only 25 numbers (the boundaries of the size sections and the corresponding concentrations) instead of the diameters of all the particles. This distribution is presented in the form of a histogram in Figure 7.1. Note that the enormous range of the aerosol particle sizes makes the presentation of the full size distribution difficult. The details of the size distribution lost by showing the whole range of diameters are illustrated in the inset of Figure 7.1.

TABLE 7.1 Example of Segregated Aerosol Size Information

Size Range (μm)	Concentration (cm^{-3})	Cumulative (cm^{-3})	Concentration ($\mu\text{m}^{-1} \text{cm}^{-3}$)
0.001–0.01	100	100	11111
0.01–0.02	200	300	20000
0.02–0.03	30	330	3000
0.03–0.04	20	350	2000
0.04–0.08	40	390	1000
0.08–0.16	60	450	750
0.16–0.32	200	650	1250
0.32–0.64	180	830	563
0.64–1.25	60	890	117
1.25–2.5	20	910	160
2.5–5.0	5	915	80
5.0–10.0	1	916	2

**FIGURE 7.1** Histogram of aerosol particle number concentrations versus the size range for the distribution of Table 7.1. The diameter range 0 to 0.2 μm for the same distribution is shown in the inset.

The size distribution of a particle population can also be described by using its cumulative distribution. The cumulative distribution value for a size section is defined as the concentration of particles that are smaller than or equal to this size range. For example, for the distribution of Table 7.1, the value of the cumulative distribution for the 0.03 to 0.04 μm size range indicates that there are 350 particles cm^{-3} that are smaller than 0.04 μm . The last value of the cumulative distribution indicates the total particle number concentration.

Use of size bins with different widths makes the interpretation of absolute concentrations difficult. For example, one may want to find out in which size range there are a lot of particles. The number concentrations in Table 7.1 indicate that there are 200 particles cm^{-3} in the range from 0.01 to 0.02 μm and another 200 particles cm^{-3} from 0.16 to 0.32 μm . However, this comparison of the concentration of particles covering a size range of 20 nm with that over a 160 nm range favors the latter. To avoid such biases, one often normalizes the distribution by dividing the concentration with the corresponding size range. The result is a concentration expressed in $\mu\text{m}^{-1} \text{cm}^{-3}$ (Table 7.1) and is illustrated in Figure 7.2. The distribution changes shape, but now the area below the curve is proportional to the number concentration. Figure 7.2 indicates that roughly half of the particles are smaller than 0.1 μm . A plot like Figure 7.1 may be misleading, as it indicates that almost all particles are larger than 0.1 μm . If a logarithmic scale is used for the diameter (Figure 7.3) both the

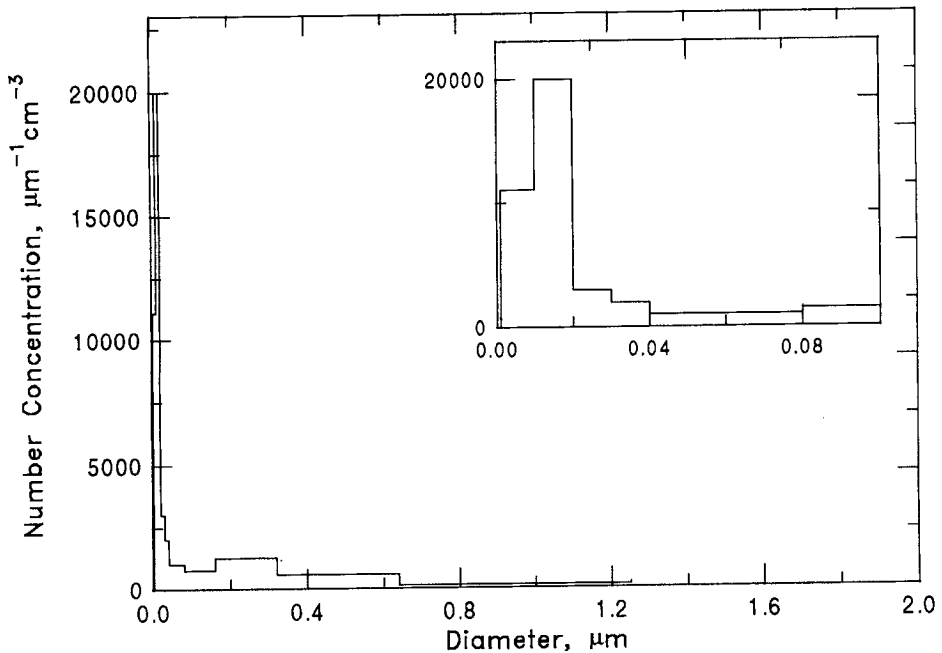


FIGURE 7.2 Aerosol number concentration normalized by the width of the size range versus size for the distribution of Table 7.1. The diameter range 0 to 0.1 μm for the same distribution is shown in the inset.

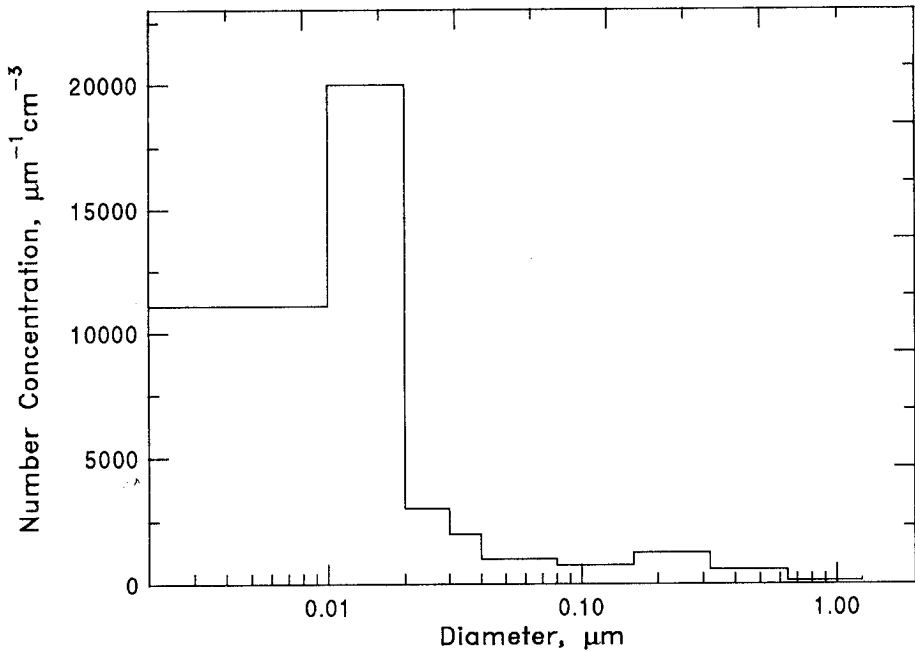


FIGURE 7.3 Same as Figure 7.2 but plotted versus the logarithm of the diameter.

large-and small-particle regions are depicted, but it now erroneously appears that the distribution consists almost exclusively of particles smaller than $0.1 \mu\text{m}$.

Using a number of size bins to describe an aerosol size distribution generally results in loss of information about the distribution structure inside each bin. While this may be acceptable for some applications, our goal in this chapter is to develop a rigorous mathematical framework for the description of the aerosol size distribution. The issues discussed in the preceding example provide valuable insights into how we should express and present ambient aerosol size distributions.

7.1.1 The Number Distribution $n_N(D_p)$

In the previous section, the value of the aerosol distribution n_i for a size interval i was expressed as the ratio of the absolute aerosol concentration N_i of this interval and the size range ΔD_p . The aerosol concentration can then be calculated by

$$N_i = n_i \Delta D_p$$

The use of arbitrary intervals ΔD_p can be confusing and makes the intercomparison of size distributions difficult. To avoid these complications and to maintain all the information regarding the aerosol distribution, one can use smaller and smaller size bins, effectively tak-

ing the limit $\Delta D_p \rightarrow 0$. At this limit, ΔD_p becomes infinitesimally small and equal to dD_p . Then one can define the size distribution function $n_N(D_p)$, as follows:

$$n_N(D_p) dD_p = \text{the number of particles per cm}^3 \text{ of air having diameters} \\ \text{in the range } D_p \text{ to } D_p + dD_p$$

The units of $n_N(D_p)$ are $\mu\text{m}^{-1} \text{cm}^{-3}$ and the total number of particles per cm^{-3} , N , is then just

$$N = \int_0^{\infty} n_N(D_p) dD_p \quad (7.1)$$

By using the function $n_N(D_p)$ we implicitly assume that the number distribution is no longer a discrete function of the number of molecules, but a continuous function of the diameter D_p . This assumption of a continuous size distribution is valid beyond a certain number of molecules, say, around 100. In the atmosphere most of the particles have diameters smaller than $0.1 \mu\text{m}$ and the number distribution function $n_N(D_p)$ usually exhibits a narrow spike near the origin (Figure 7.4).

We can define a normalized size distribution function $\bar{n}_N(D_p)$ by $\bar{n}_N(D_p) = n_N(D_p)/N$, such that

$$\bar{n}_N(D_p) dD_p = \text{the fraction of the total number of particles per cm}^3 \text{ having diameters} \\ \text{in the range } D_p \text{ to } D_p + dD_p$$

The units of $\bar{n}_N(D_p)$ are μm^{-1} . The normalized size distribution function $\bar{n}_N(D_p)$ can also be viewed as the probability that a randomly selected particle has a diameter in the range $(D_p, D_p + dD_p)$; it is therefore equivalent to the normalized probability density of particle size.

If $dN = n_N(D_p) dD_p$ denotes the number of particles in the size range $(D_p, D_p + dD_p)$, then $n_N(D_p)$ can be written as

$$n_N(D_p) = \frac{dN}{dD_p} \quad (7.2)$$

Both sides of (7.2) represent the same aerosol distribution, and the notation dN/dD_p is often used instead of $n_N(D_p)$. To conform with the common notation we will also express the distributions in this manner.

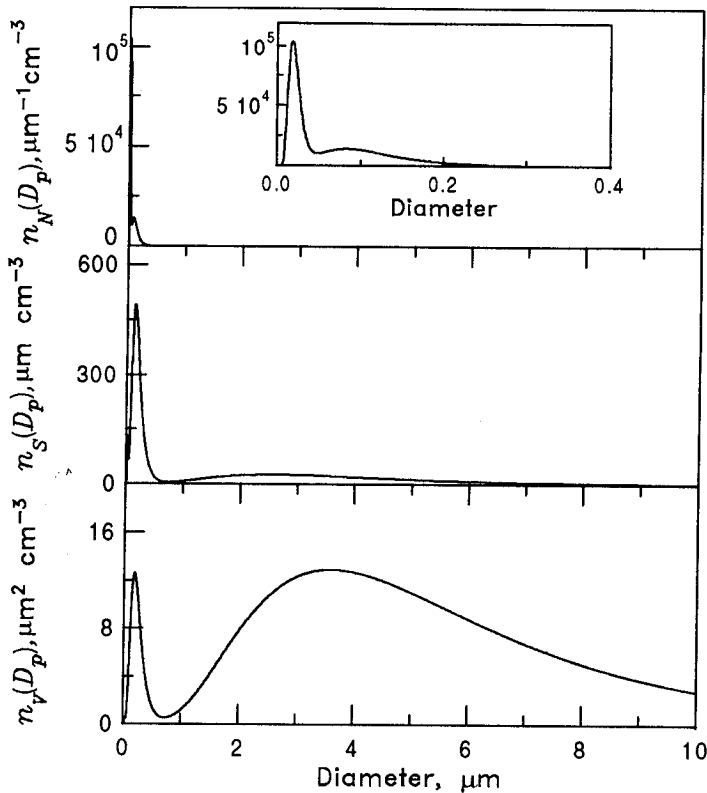


FIGURE 7.4 Atmospheric aerosol number, surface, and volume continuous distributions versus particle size. The diameter range 0 to 0.5 μm for the number distribution is shown as an inset.

Example 7.1 For the distribution of Figure 7.4, how many particles of diameter 0.1 μm exist?

According to the inset of Figure 7.4, $n_N(0.1 \mu\text{m}) = 13,000 \mu\text{m}^{-1} \text{cm}^{-3}$. However, this is not the number of particles of diameter 0.1 μm (it even has the wrong units). To calculate the number of particles we need to multiply n_N by the width of the size range ΔD_p . But if we are interested only in particles with $D_p = 0.1 \mu\text{m}$ this size range is zero and therefore there are zero particles of diameter exactly equal to 0.1 μm .

Let us try to rephrase the above question.

Example 7.2. For the distribution of Figure 7.4, how many particles with diameter in the range 0.1 to 0.11 μm exist?

The size distribution is practically constant over this narrow range with $n_N(0.1 \mu\text{m}) = 13,000 \mu\text{m}^{-1} \text{cm}^{-3}$. The width of the region is $0.11 - 0.1 = 0.01 \mu\text{m}$ and there are $0.01 \times 13,000 = 130$ particles cm^{-3} with diameters between 0.1 and 0.11 μm for this size distribution.

The above examples indicate that while n_N is a unique description of the aerosol size distribution (it does not depend on definitions of size bins, etc.), one should be careful with its physical interpretation.

7.1.2 The Surface Area, Volume, and Mass Distributions

Several aerosol properties depend on the particle surface area and volume distributions with respect to particle size. Let us define the aerosol surface area distribution $n_S(D_p)$ as

$n_S(D_p) dD_p =$ the surface area of particles per cm^3 of air having diameters
in the range D_p to $D_p + dD_p$

and let us consider all particles as spheres. All the particles in this infinitesimally narrow size range have effectively the same diameter D_p , and each of them has surface area πD_p^2 . There are $n_N(D_p) dD_p$ particles in this size range and therefore their surface area is $\pi D_p^2 n_N(D_p) dD_p$. But then by definition

$$n_S(D_p) = \pi D_p^2 n_N(D_p) \quad (\mu\text{m cm}^{-3}) \quad (7.3)$$

The total surface area S of the aerosol per cm^3 of air is then

$$S = \pi \int_0^\infty D_p^2 n_N(D_p) dD_p = \int_0^\infty n_S(D_p) dD_p \quad (\mu\text{m}^2 \text{cm}^{-3}) \quad (7.4)$$

and is equal to the area below the $n_S(D_p)$ curve in Figure 7.4. The aerosol volume distribution $n_V(D_p)$ can be defined as

$n_V(D_p) dD_p =$ the volume of particles per cm^3 of air having diameters
in the range D_p to $D_p + dD_p$

and therefore

$$n_V(D_p) = \frac{\pi}{6} D_p^3 n_N(D_p) \quad (\mu\text{m}^3 \text{cm}^{-3}) \quad (7.5)$$

The total aerosol volume per cm^3 of air, V , is

$$V = \frac{\pi}{6} \int_0^{\infty} D_p^3 n_N(D_p) dD_p = \int_0^{\infty} n_V(D_p) dD_p \quad (\mu\text{m}^3 \text{cm}^{-3}) \quad (7.6)$$

and is equal to the area below the $n_V(D_p)$ curve in Figure 7.4.

If the particles all have density ρ_p (g cm^{-3}) then the distribution of particle mass with respect to particle size, $n_M(D_p)$, is

$$n_M(D_p) = \left(\frac{\rho_p}{10^6}\right) n_V(D_p) = \left(\frac{\rho_p}{10^6}\right) \left(\frac{\pi}{6}\right) D_p^3 n_N(D_p) \quad (\mu\text{g } \mu\text{m}^{-1} \text{cm}^{-3}) \quad (7.7)$$

where the factor 10^6 is needed to convert the units of density ρ_p from g cm^{-3} to $\mu\text{g } \mu\text{m}^{-3}$, and to maintain the units for $n_M(D_p)$ as $\mu\text{g } \mu\text{m}^{-1} \text{cm}^{-3}$.

Because particle diameters in an aerosol population typically vary over several orders of magnitude, use of the distribution functions, $n_N(D_p)$, $n_S(D_p)$, $n_V(D_p)$, and $n_M(D_p)$, is often inconvenient. For example, all the structure of the number distribution depicted in Figure 7.4 occurs in the region from a few nanometers to $0.3 \mu\text{m}$ diameter, a small part of the 0 to $10 \mu\text{m}$ range of interest. To circumvent this scale problem the horizontal axis can be scaled in logarithmic intervals so that several orders of magnitude in D_p can be clearly seen (Figure 7.5). Plotting $n_N(D_p)$ on semilog axes gives, however, a somewhat dis-

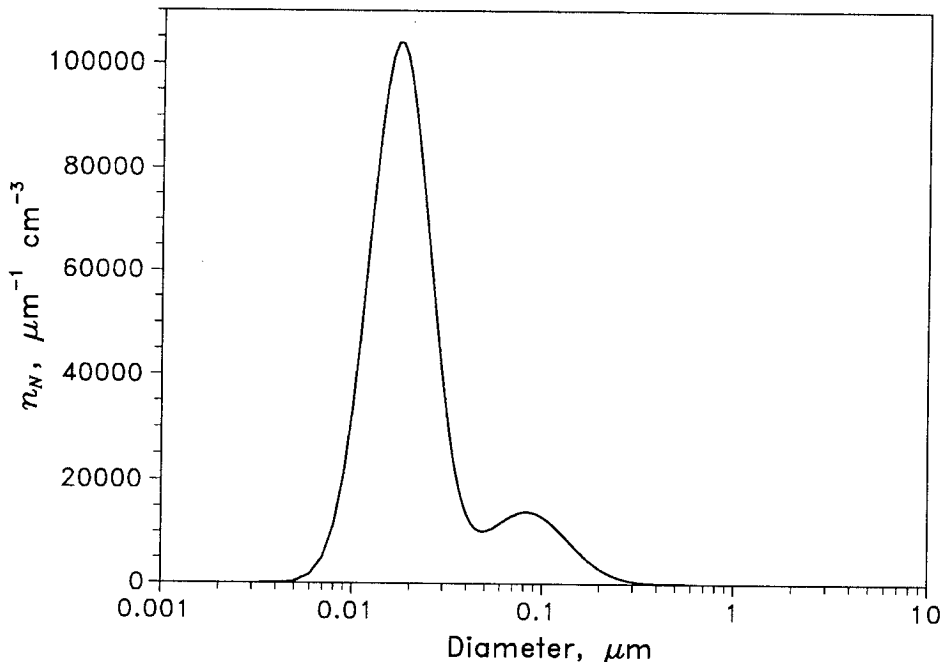


FIGURE 7.5 The same aerosol distribution as in Figure 7.4, plotted versus the logarithm of the diameter.

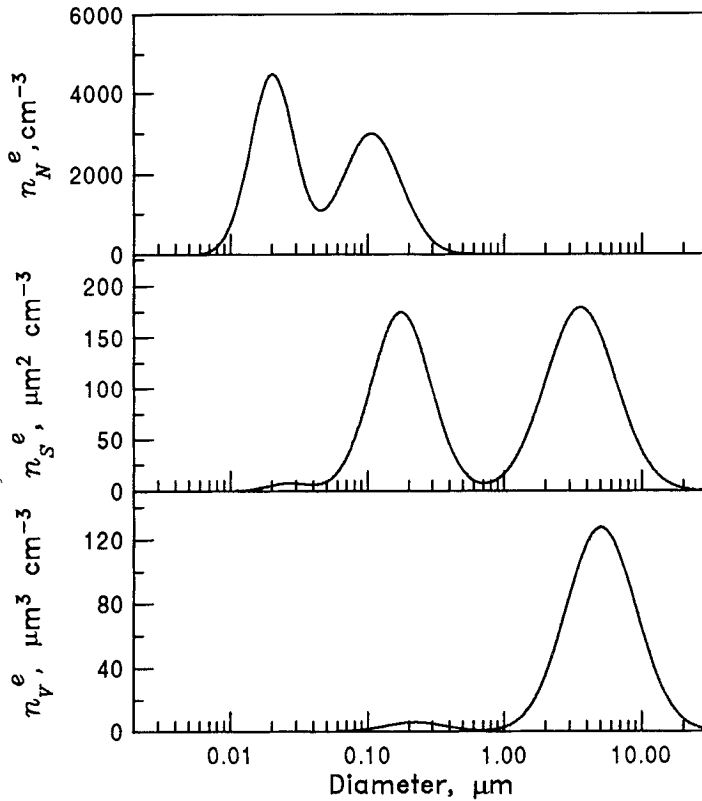


FIGURE 7.6 The same aerosol distribution as in Figures 7.4 and 7.5 expressed as a function of $\log D_p$ and plotted versus $\log D_p$. Also shown are the surface and volume distributions. The areas below the three curves correspond to the total aerosol number, surface, and volume, respectively.

torted picture of the aerosol distribution. The area below the curve no longer corresponds to the aerosol number concentration. For example, Figure 7.5 appears to suggest that more than 90% of the particles are in the smaller mode centered around $0.02 \mu\text{m}$. In reality, the numbers of particles in the two modes are almost equal (Figure 7.6).

7.1.3 Distributions Based on $\ln D_p$ and $\log D_p$

Expressing the aerosol distributions as functions of $\ln D_p$ or $\log D_p$ instead of D_p is often the most convenient way to represent the aerosol size distribution. Formally, we cannot take the logarithm of a dimensional quantity. Thus, when we write $\ln D_p$, we really mean $\ln(D_p/1)$, where the “reference” particle diameter is $1 \mu\text{m}$ and is not explicitly indicated. We can therefore define the number distribution function $n_N^e(\ln D_p)$ as

$$n_N^e(\ln D_p) d \ln D_p = \text{number of particles per cm}^3 \text{ of air in the size range} \\ \ln D_p \text{ to } \ln D_p + d \ln D_p$$

The units of $n_N^e(\ln D_p)$ are cm^{-3} since $\ln D_p$ is dimensionless. The total number concentration of particles N is

$$N = \int_{-\infty}^{\infty} n_N^e(\ln D_p) d \ln D_p \quad (\text{cm}^{-3}) \quad (7.8)$$

The limits of integration in (7.8) are from $-\infty$ to ∞ as the independent variable is $\ln D_p$.

The surface area and volume distributions as functions of $\ln D_p$ can be defined similarly to those with respect to D_p ,

$$n_S^e(\ln D_p) = \pi D_p^2 n_N^e(\ln D_p) \quad (\mu\text{m}^2 \text{cm}^{-3}) \quad (7.9)$$

$$n_V^e(\ln D_p) = \frac{\pi}{6} D_p^3 n_N^e(\ln D_p) \quad (\mu\text{m}^3 \text{cm}^{-3}) \quad (7.10)$$

with

$$\begin{aligned} S &= \pi \int_{-\infty}^{\infty} D_p^2 n_N^e(\ln D_p) d \ln D_p \\ &= \int_{-\infty}^{\infty} n_S^e(\ln D_p) d \ln D_p \quad (\mu\text{m}^2 \text{cm}^{-3}) \end{aligned} \quad (7.11)$$

$$\begin{aligned} V &= \frac{\pi}{6} \int_{-\infty}^{\infty} D_p^3 n_N^e(\ln D_p) d \ln D_p \\ &= \int_{-\infty}^{\infty} n_V^e(\ln D_p) d \ln D_p \quad (\mu\text{m}^3 \text{cm}^{-3}) \end{aligned} \quad (7.12)$$

The above aerosol distributions can also be expressed as functions of the base 10 logarithm $\log D_p$, defining $n_N^\circ(\log D_p)$, $n_S^\circ(\log D_p)$, and $n_V^\circ(\log D_p)$. Note that n_N , n_N^e , and n_N° are different mathematical functions, and, for the same diameter D_p , they have different arguments, namely D_p , $\ln D_p$, and $\log D_p$. The expressions relating these functions will be derived in the next section.

Using the notation $dN/dS/dV =$ the differential number/surface/volume of particles in the size range D_p to $D_p + dD_p$ we have

$$dN = n_N(D_p) dD_p = n_N^e(\ln D_p) d \ln D_p = n_N^\circ(\log D_p) d \log D_p \quad (7.13)$$

$$dS = n_S(D_p) dD_p = n_S^e(\ln D_p) d \ln D_p = n_S^\circ(\log D_p) d \log D_p \quad (7.14)$$

$$dV = n_V(D_p) dD_p = n_V^e(\ln D_p) d \ln D_p = n_V^\circ(\log D_p) d \log D_p \quad (7.15)$$

Based on that notation, the various size distributions are

$$\begin{aligned}
 n_N(D_p) &= \frac{dN}{dD_p} & n_N^e(\ln D_p) &= \frac{dN}{d \ln D_p} & n_N^\circ(\log D_p) &= \frac{dN}{d \log D_p} \\
 n_S(D_p) &= \frac{dS}{dD_p} & n_S^e(\ln D_p) &= \frac{dS}{d \ln D_p} & n_S^\circ(\log D_p) &= \frac{dS}{d \log D_p} \\
 n_V(D_p) &= \frac{dV}{dD_p} & n_V^e(\ln D_p) &= \frac{dV}{d \ln D_p} & n_V^\circ(\log D_p) &= \frac{dV}{d \log D_p}
 \end{aligned} \tag{7.16}$$

7.1.4 Relating Size Distributions Based on Different Independent Variables

It is often necessary to relate a size distribution based on one independent variable, say, D_p , to one based on another independent variable, say, $\log D_p$. Such a relation can be derived based on (7.13). The number of particles dN in an infinitesimal size range D_p to $D_p + dD_p$ is the same regardless of the expression used for the description of the size distribution function. Thus in the particular case of $n_N(D_p)$ and $n_N^\circ(\log D_p)$

$$n_N(D_p) dD_p = n_N^\circ(\log D_p) d \log D_p \tag{7.17}$$

Since $d \log D_p = d \ln D_p / 2.303 = dD_p / 2.303D_p$, (7.17) becomes

$$n_N^\circ(\log D_p) = 2.303 D_p n_N(D_p) \tag{7.18}$$

Similarly,

$$n_S^\circ(\log D_p) = 2.303 D_p n_S(D_p) \tag{7.19}$$

$$n_V^\circ(\log D_p) = 2.303 D_p n_V(D_p) \tag{7.20}$$

The distributions with respect to D_p are related to those with respect to $\ln D_p$ by

$$n_N^e(\ln D_p) = D_p n_N(D_p) \tag{7.21}$$

$$n_S^e(\ln D_p) = D_p n_S(D_p) \tag{7.22}$$

$$n_V^e(\ln D_p) = D_p n_V(D_p) \tag{7.23}$$

This procedure can be generalized to relate any two size distribution functions $n(u)$ and $n(v)$, where both u and v are related to D_p . The generalization of (7.17) is

$$n(u) du = n(v) dv \quad (7.24)$$

and dividing both sides by dD_p

$$n(u) = n(v) \frac{(dv/dD_p)}{(du/dD_p)} \quad (7.25)$$

7.1.5 Properties of Size Distributions

It is often convenient to summarize the features of an aerosol distribution using one or two of its properties (mean particle size, spread of distribution) than by using the full function $n_N(D_p)$. Growth of particles corresponds to a shifting of parts of the distribution to larger sizes or simply an increase of the mean particle size. These properties are called the *moments* of the distribution, and the two most often used are the mean and the variance.

Let us assume that we have a discrete distribution consisting of M groups of particles, with diameters D_k and number concentrations N_k , $k = 1, 2, \dots, M$. The number concentration of aerosols is therefore

$$N = \sum_{k=1}^M N_k \quad (7.26)$$

The mean particle diameter, \bar{D}_p , of the population is

$$\bar{D}_p = \frac{\sum_{k=1}^M N_k D_k}{\sum_{k=1}^M N_k} = \frac{1}{N} \sum_{k=1}^M N_k D_k \quad (7.27)$$

The variance, σ^2 , a measure of the spread of the distribution around the mean diameter \bar{D}_p , is defined by

$$\sigma^2 = \frac{\sum_{k=1}^M N_k (D_k - \bar{D}_p)^2}{\sum_{k=1}^M N_k} = \frac{1}{N} \sum_{k=1}^M N_k (D_k - \bar{D}_p)^2 \quad (7.28)$$

A value of σ^2 equal to zero would mean that every one of the particles in the distribution has precisely diameter \bar{D}_p . An increasing σ^2 indicates that the spread of the distribution around the mean diameter D_p is increasing.

We will usually deal with aerosol distributions in continuous form. Given the number distribution $n_N(D_p)$, (7.27) and (7.28) can be written in continuous form to define the

mean particle diameter of the distribution by

$$\bar{D}_p = \frac{\int_0^\infty D_p n_N(D_p) dD_p}{\int_0^\infty n_N(D_p) dD_p} = \frac{1}{N} \int_0^\infty D_p n_N(D_p) dD_p \quad (7.29)$$

and the variance of the distribution by

$$\sigma^2 = \frac{\int_0^\infty (D_p - \bar{D}_p)^2 n_N(D_p) dD_p}{\int_0^\infty n_N(D_p) dD_p} = \frac{1}{N} \int_0^\infty (D_p - \bar{D}_p)^2 n_N(D_p) dD_p \quad (7.30)$$

Table 7.2 presents a number of other mean values that are often used in characterizing an aerosol size distribution.

TABLE 7.2 Mean Values Often Used in Characterizing an Aerosol Size Distribution

Property	Defining Relation	Description
Number mean diameter, \bar{D}_p	$\bar{D}_p = \int_0^\infty D_p n_N(D_p) dD_p$	Average diameter of the population
Median diameter, D_{med}	$\int_0^{D_{\text{med}}} n_N(D_p) dD_p = \frac{1}{2} N$	Diameter below which one-half the particles lie and above which one-half the particles lie
Mean surface area, \bar{S}	$\bar{S} = \frac{1}{N} \int_0^\infty n_S(D_p) dD_p$	Average surface area of the population
Mean, volume, \bar{V}	$\bar{V} = \frac{1}{N} \int_0^\infty n_V(D_p) dD_p$	Average volume of the population
Surface area mean diameter, D_S	$N\pi D_S^2 = \int_0^\infty n_S(D_p) dD_p$	Diameter of the particle whose surface area equals the mean surface area of the population
Volume mean diameter, D_V	$N\frac{\pi}{6} D_V^3 = \int_0^\infty n_V(D_p) dD_p$	Diameter of the particle whose volume equals the mean volume of the population
Surface area median diameter, D_{S_m}	$\int_0^{D_{S_m}} n_S(D_p) dD_p = \frac{1}{2} \int_0^\infty n_S(D_p) dD_p$	Diameter below which one-half the particle surface area lies and above which one-half the particle surface area lies
Volume median diameter, D_{V_m}	$\int_0^{D_{V_m}} n_V(D_p) dD_p = \frac{1}{2} \int_0^\infty n_V(D_p) dD_p$	Diameter below which one-half the particle volume lies and above which one-half the particle volume lies
Mode diameter, D_{mode}	$\left(\frac{dn_N(D_p)}{dD_p} \right)_{D_{\text{mode}}} = 0$	Local maximum of the number distribution

7.1.6 The Log-Normal Distribution

A measured aerosol size distribution can be reported as a table of the distribution values for dozens of diameters. For many applications carrying around hundreds or thousands of aerosol distribution values is awkward. In these cases it is often convenient to use a relatively simple mathematical function to describe the atmospheric aerosol distribution. These functions are semiempirical in nature and have been chosen because they match well observed shapes of ambient distributions (Hinds, 1982). Of the various mathematical functions that have been proposed, the log-normal distribution (Aitchison and Brown, 1957) often provides a good fit and is regularly used in atmospheric applications. A series of other distributions are discussed in the next section.

The normal distribution for a quantity u defined from $-\infty < u < \infty$ is given by

$$n(u) = \frac{N}{(2\pi)^{1/2}\sigma_u} \exp\left(-\frac{(u - \bar{u})^2}{2\sigma_u^2}\right) \quad (7.31)$$

where \bar{u} is the mean of the distribution, σ_u^2 is the variance, and

$$N = \int_{-\infty}^{\infty} n(u) du \quad (7.32)$$

The normal distribution has the characteristic bell shape, with a maximum at \bar{u} . The standard deviation, σ_u , quantifies the width of the distribution, and 68% of the area below the curve is in the range $\bar{u} \pm \sigma_u$.

A quantity u is *log-normally distributed* if its logarithm is normally distributed. Either the natural ($\ln u$) or the base 10 logarithm ($\log u$) can be used, but since the former is more common, we will express our results in terms of $\ln D_p$. An aerosol population is therefore log-normally distributed if $u = \ln D_p$ satisfies (7.31), or

$$n_N^e(\ln D_p) = \frac{dN}{d \ln D_p} = \frac{N}{(2\pi)^{1/2} \ln \sigma_g} \exp\left(-\frac{(\ln D_p - \ln \bar{D}_{pg})^2}{2 \ln^2 \sigma_g}\right) \quad (7.33)$$

where N is the total aerosol number concentration, and \bar{D}_{pg} and σ_g are for the time being the two parameters of the distribution. Shortly we will discuss the physical significance of these parameters. The distribution $n_N(D_p)$ is often used instead of $n_N^e(\ln D_p)$. Combining (7.21) with (7.33)

$$n_N(D_p) = \frac{dN}{dD_p} = \frac{N}{(2\pi)^{1/2} D_p \ln \sigma_g} \exp\left(-\frac{(\ln D_p - \ln \bar{D}_{pg})^2}{2 \ln^2 \sigma_g}\right) \quad (7.34)$$

A log-normal aerosol distribution with $\bar{D}_{pg} = 0.8 \mu\text{m}$ and $\sigma_g = 1.5$ is depicted in Figure 7.7.

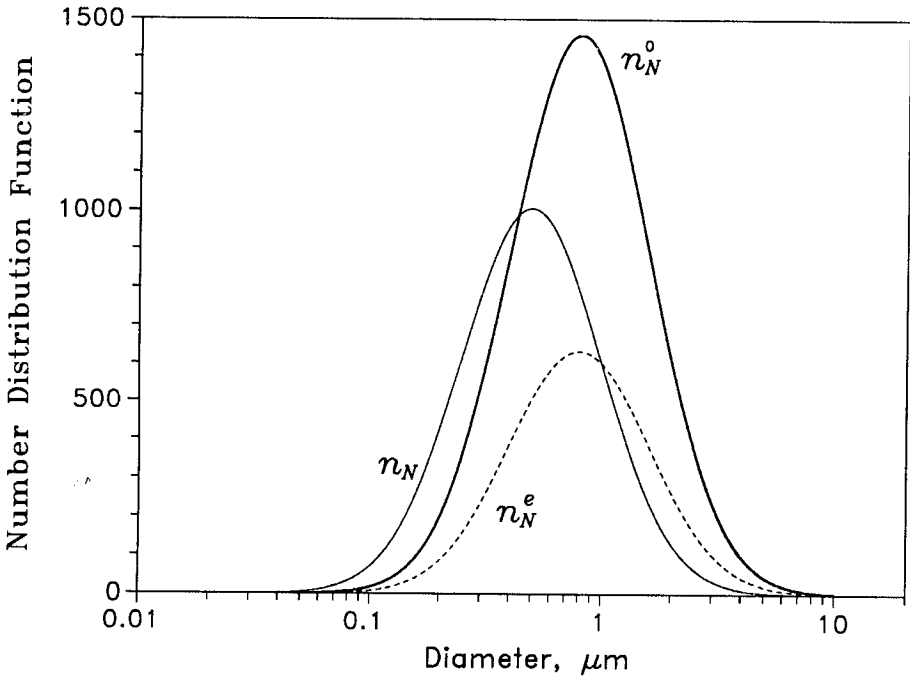


FIGURE 7.7 Aerosol distribution functions, $n_N(D_p)$, $n_N^o(\log D_p)$ and $n_N^e(\ln D_p)$ for a log-normally distributed aerosol distribution $D_{pg} = 0.8 \mu\text{m}$ and $\sigma_g = 1.5$ versus $\log D_p$. Even if all three functions describe the same aerosol population, they differ from each other because they use a different independent variable. The aerosol number is the area below the $n_N^o(\log D_p)$ curve.

We now wish to examine the physical significance of the two parameters \bar{D}_{pg} and σ_g . To do so let us define the cumulative size distribution $F_N(D_p^*)$ as the concentration of particles in the population with diameters smaller than or equal to D_p^* , that is,

$$F_N(D_p^*) = \int_0^{D_p^*} n_N(D_p) dD_p \tag{7.35}$$

If the aerosol distribution is log-normal, $n_N(D_p)$ is given by (7.34) and therefore

$$F_N(D_p^*) = \frac{N}{(2\pi)^{1/2} \ln \sigma_g} \int_0^{D_p^*} \frac{1}{D_p} \exp \left[-\frac{(\ln D_p - \ln \bar{D}_{pg})^2}{2 \ln^2 \sigma_g} \right] dD_p \tag{7.36}$$

To evaluate this integral we let $\eta = (\ln D_p - \ln \bar{D}_{pg})/\sqrt{2} \ln \sigma_g$ and we obtain

$$F_N(D_p^*) = \frac{N}{\sqrt{\pi}} \int_{-\infty}^{(\ln D_p^* - \ln \bar{D}_{pg})/\sqrt{2} \ln \sigma_g} e^{-\eta^2} d\eta \tag{7.37}$$

The error function $\operatorname{erf} z$ is defined as

$$\operatorname{erf} z = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\eta^2} d\eta \tag{7.38}$$

and $\operatorname{erf}(0) = 0, \operatorname{erf}(\infty) = 1$. If we divide the integral in (7.37) into one from $-\infty$ to 0 and the second from 0 to $(\ln D_p^* - \ln \bar{D}_{pg})/\sqrt{2} \ln \sigma_g$, then the first integral is seen to be equal to $\sqrt{\pi}/2$ and the second to $(\sqrt{\pi}/2)\operatorname{erf}[(\ln D_p^* - \ln \bar{D}_{pg})/\sqrt{2} \ln \sigma_g]$. Thus for the log-normal distribution

$$F_N(D_p) = \frac{N}{2} + \frac{N}{2} \operatorname{erf}\left(\frac{\ln(D_p/\bar{D}_{pg})}{\sqrt{2} \ln \sigma_g}\right) \tag{7.39}$$

For $D_p = \bar{D}_{pg}$, since $\operatorname{erf}(0) = 0$

$$F(\bar{D}_{pg}) = \frac{N}{2} \tag{7.40}$$

and we see that $\bar{D}_{pg} = D_{\text{med}}$ is the *median diameter*, that is, the diameter for which exactly one-half of the particles are smaller and one-half are larger. To understand the role of σ_g let us consider the diameter $D_{p\sigma}$ for which $\sigma_g = D_{p\sigma}/\bar{D}_{pg}$. At that diameter, using (7.39),

$$F(D_{p\sigma}) = N \left[\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] = 0.841N \tag{7.41}$$

Thus σ_g is the ratio of the diameter below which 84.1% of the particles lie to the median diameter and is termed the *geometric standard deviation*. A monodisperse aerosol population has $\sigma_g = 1$. For any distribution, 67% of all particles lie in the range from \bar{D}_{pg}/σ_g to $\bar{D}_{pg}\sigma_g$ and 95% of all particles lie in the range from $\bar{D}_{pg}/2\sigma_g$ to $2\bar{D}_{pg}\sigma_g$.

Let us calculate the mean diameter \bar{D}_p of a log-normally distributed aerosol. By definition, the mean diameter is found from

$$\bar{D}_p = \frac{1}{N} \int_0^\infty D_p n_N(D_p) dD_p \tag{7.42}$$

which we wish to evaluate in the case of $n_N(D_p)$ given by (7.34). Therefore

$$\bar{D}_p = \frac{1}{\sqrt{2\pi} \ln \sigma_g} \int_0^\infty \exp\left(-\frac{(\ln D_p - \ln \bar{D}_{pg})^2}{2 \ln^2 \sigma_g}\right) dD_p \tag{7.43}$$

After evaluating the integral one finds that

$$\bar{D}_p = \bar{D}_{pg} \exp\left(\frac{\ln^2 \sigma_g}{2}\right) \tag{7.44}$$

We see that the mean diameter of a log-normal distribution depends on both \bar{D}_{pg} and σ_g .

7.1.7 Plotting the Log-Normal Distribution

The cumulative distribution function $F_N(D_p)$ for a log-normally distributed aerosol population is given by (7.39). Defining the normalized cumulative distribution,

$$\bar{F}_N(D_p) = \frac{F_N(D_p)}{N} \quad (7.45)$$

one obtains

$$\bar{F}_N(D_p) = \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\ln D_p - \ln \bar{D}_{pg}}{\sqrt{2} \ln \sigma_g} \right) \quad (7.46)$$

The cumulative distribution fraction $\bar{F}_N(D_p)$ can be plotted against the logarithm of particle diameter on special log-probability graph paper. In these diagrams the x axis is logarithmic and the y axis is scaled according to the error function. This scaling compresses the scale near the median (50% point) and expands the scale near the ends. Several computer graphics programs also allow the use of a probability axis. In these graphs the cumulative distribution function of a log-normal distribution is a straight line (Figure 7.8). The point at $\bar{F}_N(D_p) = 0.5$ occurs when $D_p = \bar{D}_{pg}$. Therefore the geometric mean, or median, of the distribution is the value of D_p where the straight line plot of \bar{F}_N crosses the 50th percentile. The point at $\bar{F}_N(D_p) = 0.84$ occurs for $\ln D_p = \ln \bar{D}_p + \ln \sigma_g$ or $D_p = \bar{D}_p \sigma_g$. The slope of the line is therefore related to the geometric standard deviation of the distribution. Log-normal distributions with the same standard deviation when plotted in log-probability coordinates are parallel to each other. A small standard deviation corresponds to a

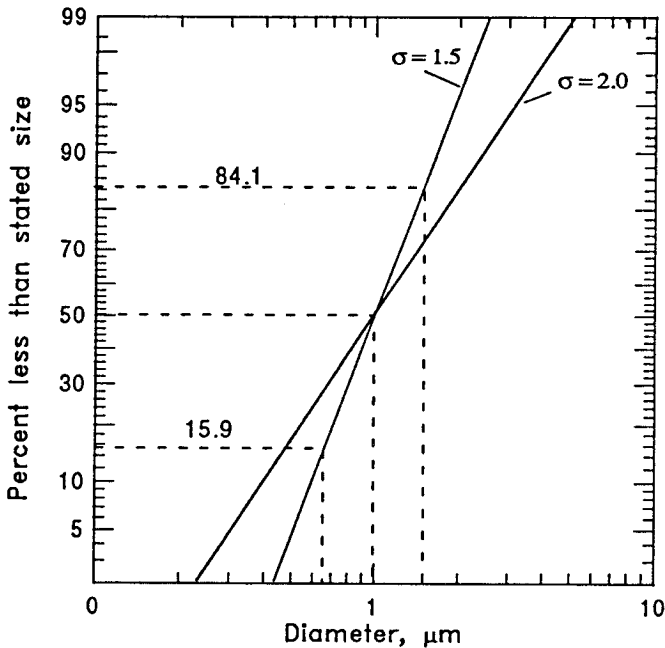


FIGURE 7.8 Cumulative log-normal aerosol number distributions plotted on log-probability paper. The distributions have mean diameter of $1 \mu\text{m}$ and $\sigma_g = 2$ and 1.5 , respectively.

narrow distribution and to a steep line in the log-probability graph (Figure 7.8). The geometric standard deviation can be calculated as the ratio of the diameter $D_{p+\sigma}$ for which $\bar{F}_N(D_{p+\sigma}) = 0.84$ to the mean diameter

$$\sigma_g = \frac{D_{p+\sigma}}{D_p} \quad (7.47)$$

7.1.8 Properties of the Log-Normal Distribution

We have discussed the properties of the log-normal distribution for the number concentration. The next step is examination of the surface and volume distributions corresponding to a log-normal number distribution given by (7.34). Since $n_S(D_p) = \pi D_p^2 n_N(D_p)$ and $n_V(D_p) = (\pi/6) D_p^3 n_N(D_p)$, let us determine the forms of $n_S(D_p)$ and $n_V(D_p)$ when $n(D_p)$ is log-normal. From (7.34) one gets

$$n_S(D_p) = \frac{\pi D_p^2 N}{(2\pi)^{1/2} D_p \ln \sigma_g} \exp\left(-\frac{(\ln D_p - \ln \bar{D}_{pg})^2}{2 \ln^2 \sigma_g}\right) \quad (7.48)$$

By letting $D_p^2 = \exp(2 \ln D_p)$, expanding the exponential, and completing the square in the exponent, (7.48) becomes

$$\begin{aligned} n_S(D_p) &= \frac{N}{(2\pi)^{1/2} D_p \ln \sigma_g} \exp(2 \ln \bar{D}_{pg} + 2 \ln^2 \sigma_g) \\ &\quad \times \exp\left(-\frac{[\ln D_p - (\ln \bar{D}_{pg} + 2 \ln^2 \sigma_g)]^2}{2 \ln^2 \sigma_g}\right) \end{aligned} \quad (7.49)$$

Thus we see that if the number distribution $n_N(D_p)$ is log-normal, the surface distribution $n_S(D_p)$ is also log-normal with the same geometric standard deviation σ_g as the parent distribution and with the surface median diameter given by

$$\ln \bar{D}_{pgS} = \ln \bar{D}_{pg} + 2 \ln^2 \sigma_g \quad (7.50)$$

The above calculations can be repeated for the volume distribution and one can show that

$$n_V(D_p) = \frac{\pi D_p^3 N}{6(2\pi)^{1/2} D_p \ln \sigma_g} \exp\left(-\frac{(\ln D_p - \ln \bar{D}_{pg})^2}{2 \ln^2 \sigma_g}\right) \quad (7.51)$$

or by letting $D_p^3 = \exp(3 \ln D_p)$, expanding the exponential, and completing the square in the exponent, (7.51) becomes

$$\begin{aligned} n_V(D_p) &= \frac{N}{(2\pi)^{1/2} D_p \ln \sigma_g} \exp(3 \ln \bar{D}_{pg} + \frac{9}{2} \ln^2 \sigma_g) \\ &\quad \times \exp\left(-\frac{[\ln D_p - (\ln \bar{D}_{pg} + 3 \ln^2 \sigma_g)]^2}{2 \ln^2 \sigma_g}\right) \end{aligned} \quad (7.51a)$$

Therefore if the number distribution $n_N(D_p)$ is log-normal, the volume distribution $n_V(D_p)$ is also log-normal with the same geometric standard deviation σ_g as the parent distribution and with the volume median diameter given by

$$\ln \bar{D}_{pgV} = \ln \bar{D}_{pg} + 3 \ln^2 \sigma_g \quad (7.52)$$

The constant standard deviation for the number, surface, and volume distributions for any log-normal distribution is one of the great advantages of this mathematical representation.

Plotting the surface and volume distributions of a log-normal aerosol distribution on log-probability paper would also result in straight lines parallel to each other (same standard deviation). For the distribution shown in Figure 7.8 with $\bar{D}_{pg} = 1.0 \mu\text{m}$ and $\sigma_g = 2.0$, the resulting surface area and volume median diameters are approximately $2.6 \mu\text{m}$ and $4.2 \mu\text{m}$, respectively.

7.1.9 Other Aerosol Distributions

The Power-Law Distribution A series of other mathematical functions have been proposed for the description of atmospheric aerosol distributions. The power law, or Junge, distribution has often been used in atmospheric science (Pruppacher and Klett, 1980)

$$n_N^\circ(\log D_p) = \frac{C}{(D_p)^\alpha} \quad (7.53)$$

where C and α are constants. Plotting of the power-law distribution on log-log coordinates results in a straight line with slope $-\alpha$ and for $D_p = 1 \mu\text{m}$, $n_N^\circ = C$ (Figure 7.9). This distribution function assumes that the aerosol number concentration decreases monotonically with increasing particle size. This is not generally true in the atmosphere so the power-law distribution should be used with caution and only for specific size ranges (usually for $D_p > 0.1 \mu\text{m}$). The derived distributions are accurate only over a limited size range and extrapolation to smaller or larger sizes may introduce significant errors. Values of α from 2 to 5 have been suggested for ambient aerosol distributions (Pruppacher and Klett, 1980) (Figure 7.9). The corresponding volume distribution can be calculated using (7.10),

$$n_V^\circ(\log D_p) = \frac{\pi C}{6} D_p^{3-\alpha} \quad (7.54)$$

Leitch and Isaac (1991) proposed that, to a first approximation, most of the number distributions for the continental aerosol distributions can be represented by a -3 power-law function. However, important features of these distributions are clearly lost by such representation. On a log-diameter volume distribution plot, the corresponding -3 power-law

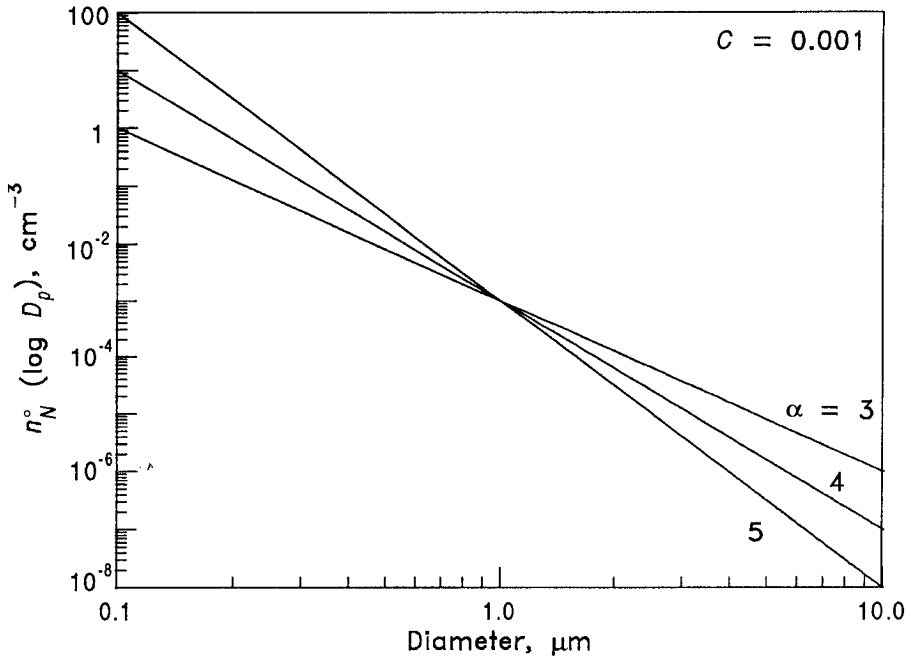


FIGURE 7.9 Power-law distributions for $C = 0.001$, and different values of α .

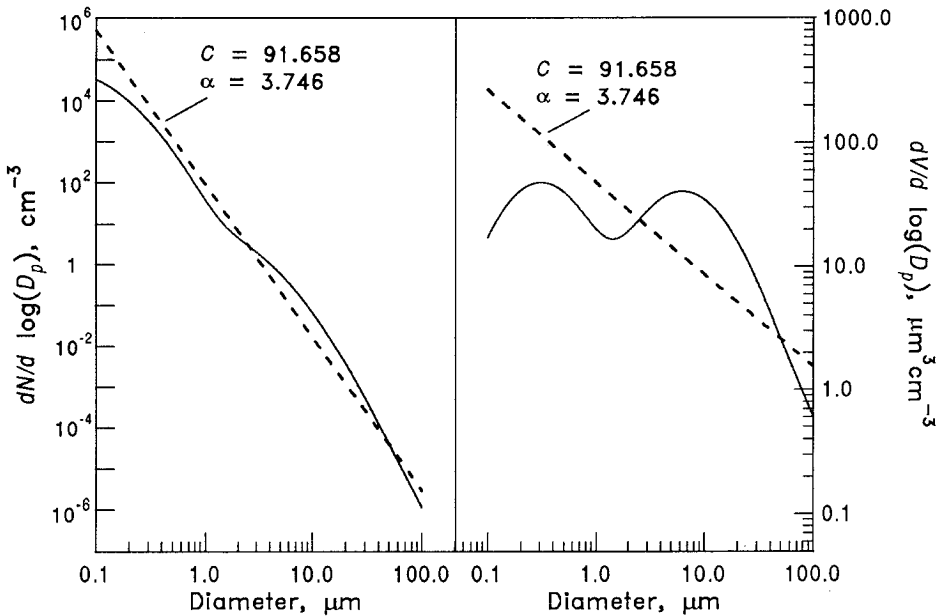


FIGURE 7.10 Fitting of an urban aerosol number distribution with a power-law distribution (left) and comparison of the corresponding volume distributions (right). Even if the power-law distribution appears to match the number distribution, it fails to reproduce the volume distribution.

function is just a straight line, yet none of the measured volume distributions was straight. This problem is depicted in Figure 7.10. A power-law distribution is fitted to urban aerosol size distribution data and it appears to provide a reasonable approximation. However, when the power-law distribution is integrated for the calculation of the volume distribution (Figure 7.10), it grossly overpredicts the volume of the submicron particles and seriously underpredicts the volume of the coarse aerosol. Therefore use of a fitted power-law distribution for the calculation of aerosol properties that depend on powers of the diameter (e.g., optical properties, condensation rates) should be avoided.

The main advantage of the power-law distribution is simplicity, but it is often inadequate for the description of ambient aerosol distributions that have significant structure. Power-law expressions can provide reasonable fits to parts of the atmospheric aerosol number distribution but they are inadequate models of the surface and volume distributions.

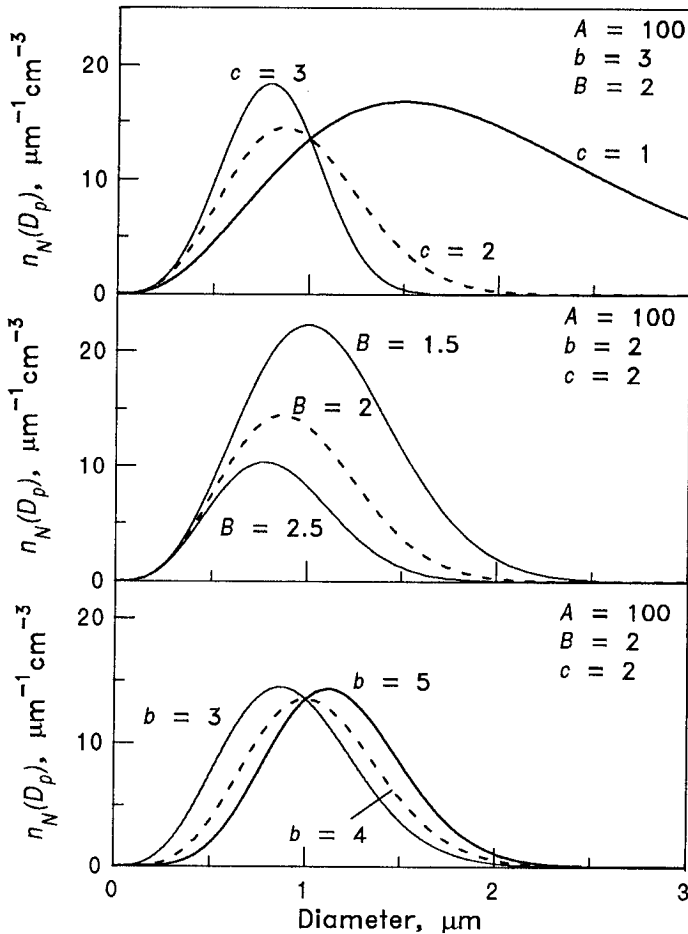


FIGURE 7.11 Modified gamma aerosol distributions for various combinations of the parameters A , b , B , and c .

The Modified Gamma Distribution The modified gamma distribution (Deirmendjian, 1969) has been proposed as another function that approximates ambient aerosol size distributions,

$$n_N(D_p) = AD_p^b \exp(-BD_p^c) \quad (7.55)$$

where A , b , B , and c are all positive parameters. This distribution form provides significant flexibility (Figure 7.11), but its use is often cumbersome. The total aerosol number concentration N is equal to

$$N = \frac{AB^{-(b+1)/c}}{c} \Gamma\left(\frac{b+1}{c}\right) \quad (7.56)$$

where Γ is the gamma function. The maximum of the distribution occurs at diameter D_m ,

$$D_m = \left(\frac{b}{Bc}\right)^{1/c} \quad (7.57)$$

7.2 AMBIENT AEROSOL SIZE DISTRIBUTIONS

Atmospheric aerosol size distributions are often described as the sum of n log-normal distributions,

$$n_N^\circ(\log D_p) = \sum_{i=1}^n \frac{N_i}{(2\pi)^{1/2} \log \sigma_i} \exp\left(-\frac{(\log D_p - \log \bar{D}_{pi})^2}{2 \log^2 \sigma_i}\right) \quad (7.58)$$

where N_i is the number concentration, \bar{D}_{pi} is the mean diameter, and σ_i is the standard deviation of the i^{th} log-normal mode. In this case $3n$ parameters are necessary for the description of the full aerosol distribution. Characteristics of model aerosol distributions are presented in Table 7.3 following the suggestions of Jaenicke (1993).

7.2.1 Urban Aerosols

Urban aerosols are mixtures of primary particulate emissions from industries, transportation, power generation, and natural sources and secondary material formed by gas-to-particle conversion mechanisms. The number distribution is dominated by particles smaller than 0.1 μm , while most of the surface area is in the 0.1 to 0.5 μm size range. On the contrary, the aerosol mass distribution has usually two distinct modes, one in the submicron regime (referred to as the accumulation mode) and the other in the coarse particle regime (Figure 7.12).

The aerosol size distribution is quite variable in an urban area. Extremely high concentrations of fine particles (less than 0.1 μm in diameter) are found close to sources (e.g., highways), but their concentration decreases rapidly with distance from the source (Figure

TABLE 7.3 Parameters for Model Aerosol Distributions Expressed as the Sum of Three Log-Normal Modes

Type	Mode I			Mode II			Mode III		
	N (cm^{-3})	D_p (μm)	$\log \sigma$	N (cm^{-3})	D_p (μm)	$\log \sigma$	N (cm^{-3})	D_p (μm)	$\log \sigma$
Urban	9.93×10^4	0.013	0.245	1.11×10^3	0.014	0.666	3.64×10^4	0.05	0.337
Marine	133	0.008	0.657	66.6	0.266	0.210	3.1	0.58	0.396
Rural	6650	0.015	0.225	147	0.054	0.557	1990	0.084	0.266
Remote continental	3200	0.02	0.161	2900	0.116	0.217	0.3	1.8	0.380
Free troposphere	129	0.007	0.645	59.7	0.250	0.253	63.5	0.52	0.425
Polar	21.7	0.138	0.245	0.186	0.75	0.300	3×10^{-4}	8.6	0.291
Desert	726	0.002	0.247	114	0.038	0.770	0.178	21.6	0.438

Source: Jaenicke (1993).

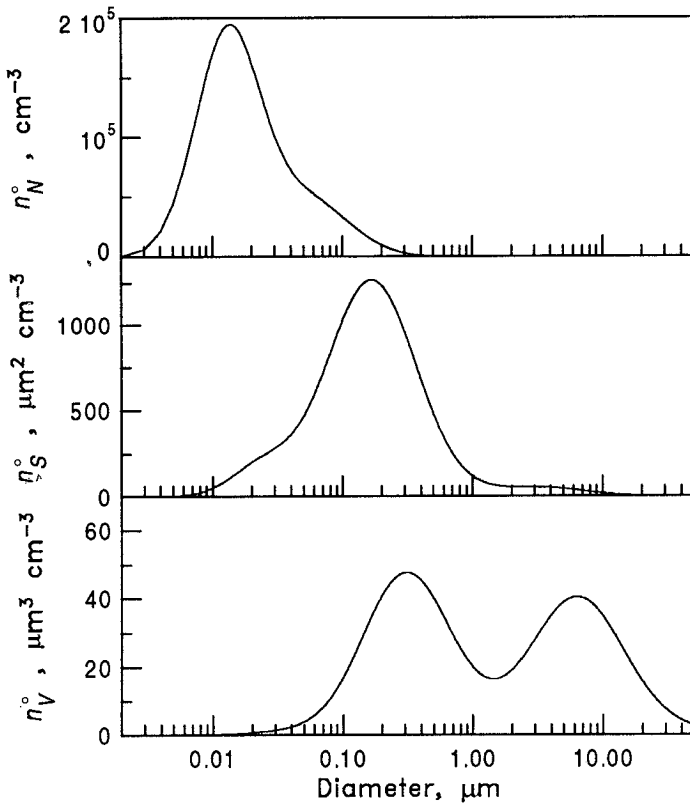


FIGURE 7.12 Typical urban aerosol number, surface, and volume distributions.

7.13). Figure 7.13 describes the number of particles as a function of their diameter (both in logarithmic scales) for a variety of environments. There are roughly an order of magnitude more particles close to the freeway compared to the average urban concentration. Figure 7.14 illustrates the corresponding volume distributions. These distributions show that most of the particles in an urban area are smaller than $0.1 \mu\text{m}$, while most of the particle mass is found in particles with diameters larger than $0.1 \mu\text{m}$.

An important feature of atmospheric aerosol size distributions is their multimodal character. Mass distributions, measured in urban centers, are characterized by three modes with a minimum between 1.0 and $3 \mu\text{m}$. The size range of particles larger than the minimum (supermicron particles) is termed "coarse," while the smaller particles are called "fine." The three modes present in the mass distribution of Figure 7.14 correspond to the nuclei mode (particles below $0.1 \mu\text{m}$), accumulation mode ($0.1 < D_p < 1 \mu\text{m}$), and coarse mode ($D_p > 1 \mu\text{m}$) (Whitby and Sverdrup, 1980). Thus the fine particles include both accumulation and nuclei modes. The boundaries between these sections are not precise (recall in Chapter 2 that we divided fine and coarse modes at $2.5 \mu\text{m}$ diameter). Note that our definition of modes has been based on the mass (or volume distribution). The location of modes may be different if they are based on the number or surface distribution.

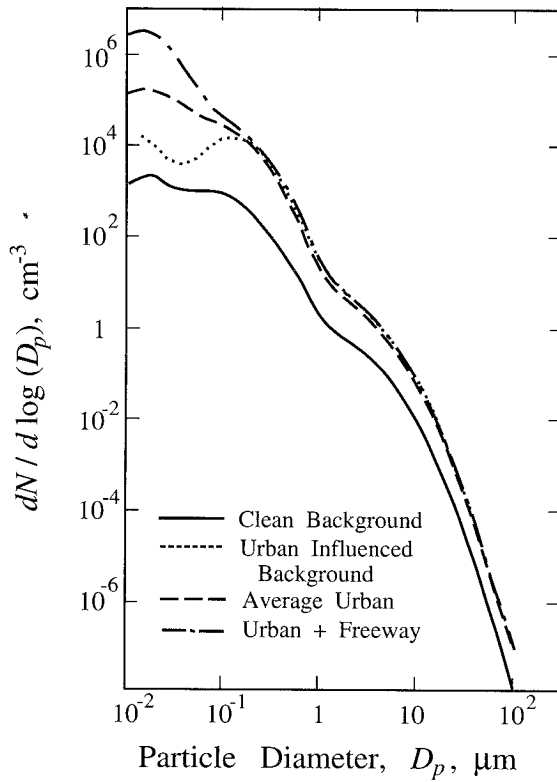


FIGURE 7.13 Aerosol number distributions next to a source (freeway), for average urban, for urban influenced background, and for background conditions.

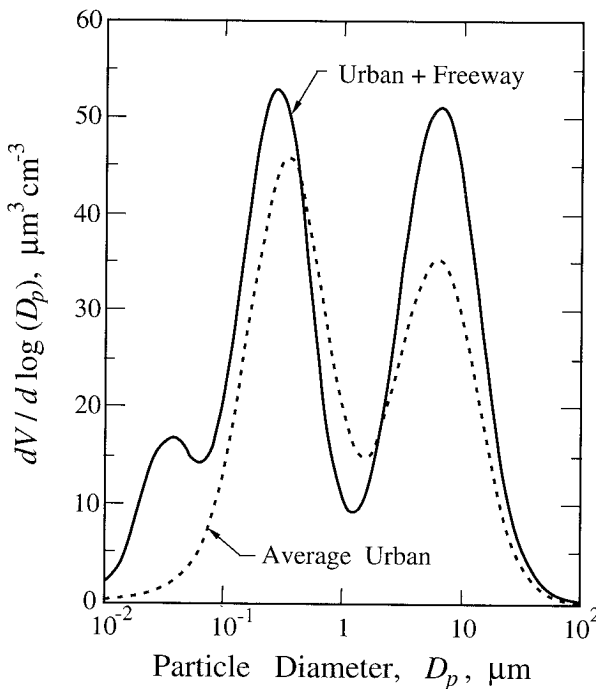


FIGURE 7.14 Aerosol volume distributions next to a source (freeway) and for average urban conditions.

The mass concentrations of the accumulation and coarse particle modes are comparable for most urban areas. The nuclei mode, with the exception of areas close to combustion sources, contains negligible volume (Figures 7.12 and 7.14). Most of the aerosol surface area is in particles of diameters 0.1 to 0.5 μm in the accumulation mode (Figure 7.12). Because of this availability of area, transfer of material from the gas phase during gas-to-particle conversion occurs preferentially on them.

The sources and chemical compositions of the fine and coarse particles are different. Coarse particles are generated by mechanical processes and consist of soil dust, sea salt, fly ash, tire wear particles, and so on. Nuclei and accumulation mode particles contain primary particles from combustion sources and secondary aerosol material (sulfate, nitrate, ammonium, secondary organics) formed by chemical reactions resulting in gas-to-particle conversion (see Chapters 9 and 13).

The main mechanisms of transfer of particles from the nuclei to accumulation mode is coagulation (Chapter 12) and growth by condensation of vapors formed by chemical reactions (Chapter 11) onto existing particles. Coagulation among accumulation mode particles is a slow process and does not transfer particles to the coarse mode.

Processing of accumulation and coarse mode aerosols by clouds (Chapter 15) can also modify the concentration and composition of these modes. Aqueous-phase chemical reactions take place in cloud and fog droplets, and in aerosol particles at relative humidities approaching 100%. These reactions can lead to production of sulfate (Chapter 6) and after evaporation of water, a larger aerosol particle is left in the atmosphere. This transformation can lead to the formation of two modes in the 0.1 to 1 μm size range, with the smaller one called the *condensation mode* and the larger one the *droplet mode* (Hering and Friedlander, 1982; John et al., 1990; Meng and Seinfeld, 1994).

Terms often used to describe the aerosol mass concentration include total suspended particulate matter (TSP) and PM_x (particulate matter with diameter smaller than x μm). TSP refers to the mass concentration of atmospheric particles smaller than 40 to 50 μm , while $\text{PM}_{2.5}$ and PM_{10} are routinely monitored. For a description of the sampling issues and problems related to the measurement of TSP, $\text{PM}_{2.5}$, and PM_{10} the reader is referred to the EPA Particulate Matter Criteria document (U.S. EPA, 1996).

7.2.2 Marine Aerosols

In the absence of significant transport of continental aerosols, particles over the remote oceans are largely of marine origin (Savoie and Prospero, 1989). Marine atmospheric particle concentrations are normally in the range of 100 to 300 cm^{-3} . Their size distribution is usually characterized by three modes (Figure 7.15): the nuclei ($D_p < 0.1$ μm) the accumulation ($0.1 < D_p < 0.6$ μm), and the coarse ($D_p > 0.6$ μm) (Fitzgerald, 1991). Typically, the coarse particle mode, comprising 95% of the total mass but only 5 to 10% of the particle number (Figure 7.16), results from the evaporation of sea spray produced by bursting bubbles or wind-induced wave breaking (Blanchard and Woodcock, 1957; Monahan et al., 1983). Typical sea-salt aerosol concentrations in the marine boundary layer (MBL) are around 5 to 30 cm^{-3} (Blanchard and Cipriano, 1987; O'Dowd and Smith, 1993).

Figures 7.15 and 7.16 show number and volume aerosol distributions in clean maritime air measured by several investigators (Mészáros and Vissy, 1974; Hoppel et al., 1989; Haaf and Jaenicke, 1980; De Leeuw, 1986) and a model marine aerosol size distribution. The dis-

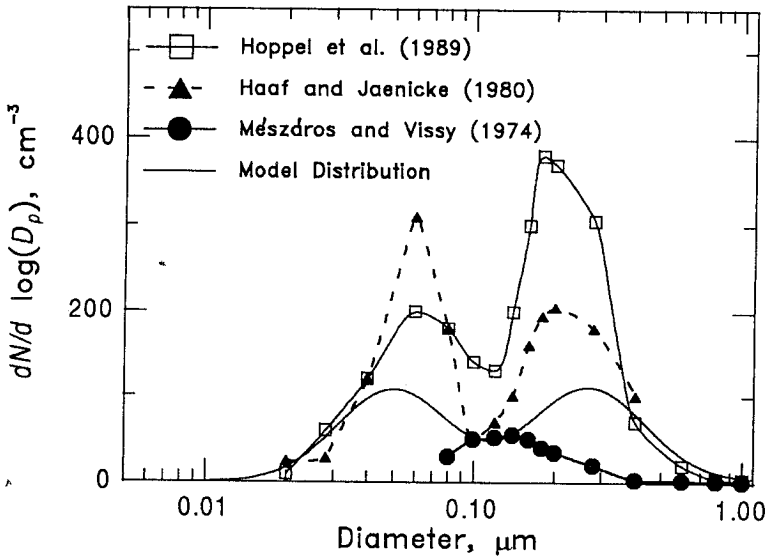


FIGURE 7.15 Measured marine aerosol number distributions and a model distribution used to represent average conditions.

tributions of Hoppel et al. (1989) and De Leeuw (1986) were obtained at wind speeds of less than 5 m s^{-1} in the subtropical and North Atlantic, respectively. The distribution of Mészáros and Vissy (1974) is an average of spectra obtained in the South Atlantic and Indian Oceans during periods when the average wind speed was 12 m s^{-1} . It is difficult to determine the extent to which the differences in these size distributions are the result of differences in sampling location and meteorological conditions such as wind speed (which af-

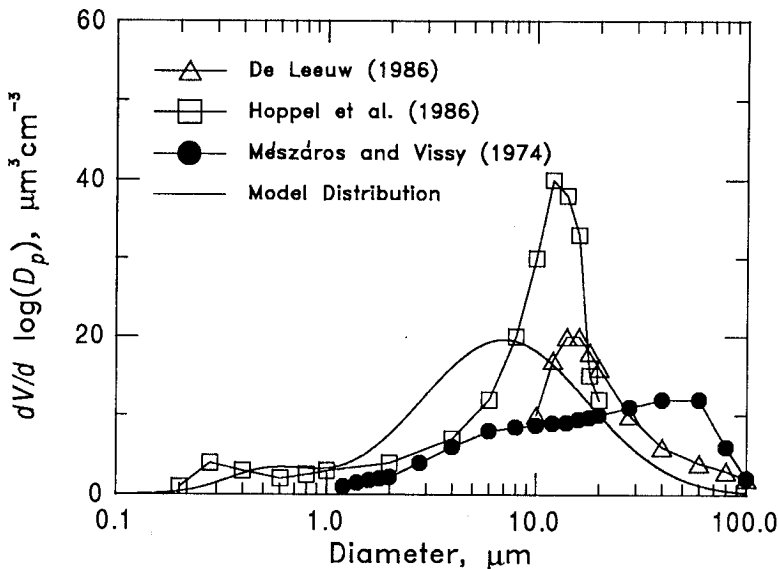


FIGURE 7.16 Measured marine aerosol volume distributions and a model distribution used to represent average conditions.

fects the concentrations of the larger particles), or to uncertainties inherent in the different measurement methods.

7.2.3 Rural Continental Aerosols

Aerosols in rural areas are mainly of natural origin but with a moderate influence of anthropogenic sources (Hobbs et al., 1985). The number distribution is characterized by two modes at diameters about 0.02 and 0.08 μm , respectively (Jaenicke, 1993), while the mass distribution is dominated by the coarse mode centered at around 7 μm (Figure 7.17). The mass distribution of continental aerosol not influenced by local sources has a small accumulation mode and no nuclei mode. The PM_{10} concentration of rural aerosols is around $20 \mu\text{g m}^{-3}$.

7.2.4 Remote Continental Aerosols

Primary particles (e.g., dust, pollens, plant waxes) and secondary oxidation products are the main components of remote continental aerosol (Deepak and Gali, 1991). Aerosol number concentrations average around 2000 to 10,000 cm^{-3} and PM_{10} concentrations are

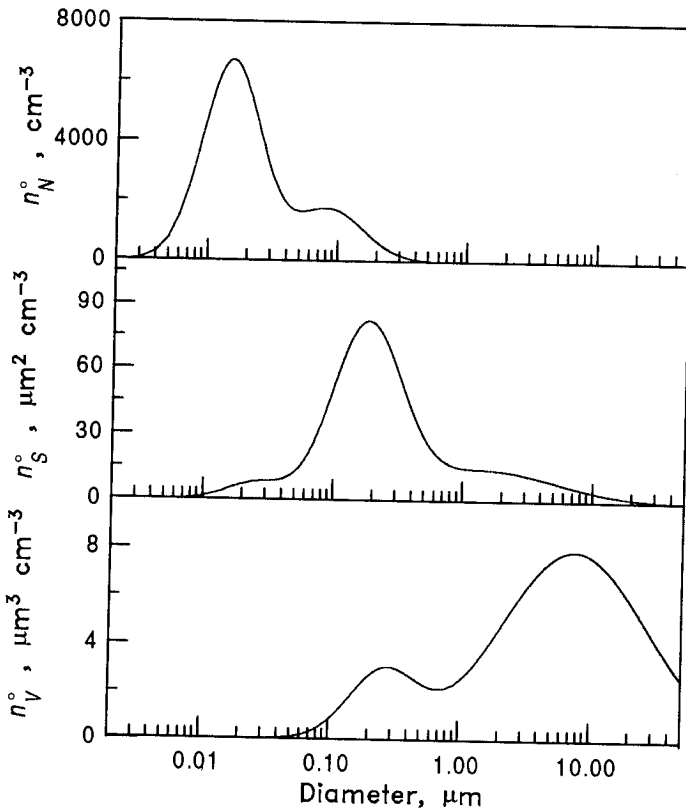


FIGURE 7.17 Typical rural continental aerosol number, surface, and volume distributions.

around $10 \mu\text{g m}^{-3}$ (Bashurova et al., 1992; Koutsenogii et al., 1993; Koutsenogii and Jaenicke, 1994). For the continental United States PM_{10} concentrations in remote areas vary from 5 to $25 \mu\text{g m}^{-3}$ and $\text{PM}_{2.5}$ from 3 to $17 \mu\text{g m}^{-3}$ (U.S. EPA, 1996). Particles smaller than $2.5 \mu\text{m}$ in diameter represent 40 to 80% of the PM_{10} mass and consist mainly of sulfate, ammonium, and organics. The aerosol number distribution may be characterized by three modes at diameters 0.02, 0.1, and $2 \mu\text{m}$ (Jaenicke, 1993) (Figure 7.18).

7.2.5 Free Tropospheric Aerosols

Background free tropospheric aerosol is found in the mid- and upper troposphere above the clouds. Although it occupies a significant fraction of the tropospheric volume, it has received relatively little attention. Most measurements have been carried out at high-elevation ground sites or in subsiding airmasses reflecting midtropospheric conditions. The modes in the number distribution correspond to mean diameters of 0.01 and $0.25 \mu\text{m}$ (Jaenicke, 1993) (Figure 7.19). The middle troposphere spectra typically indicate more

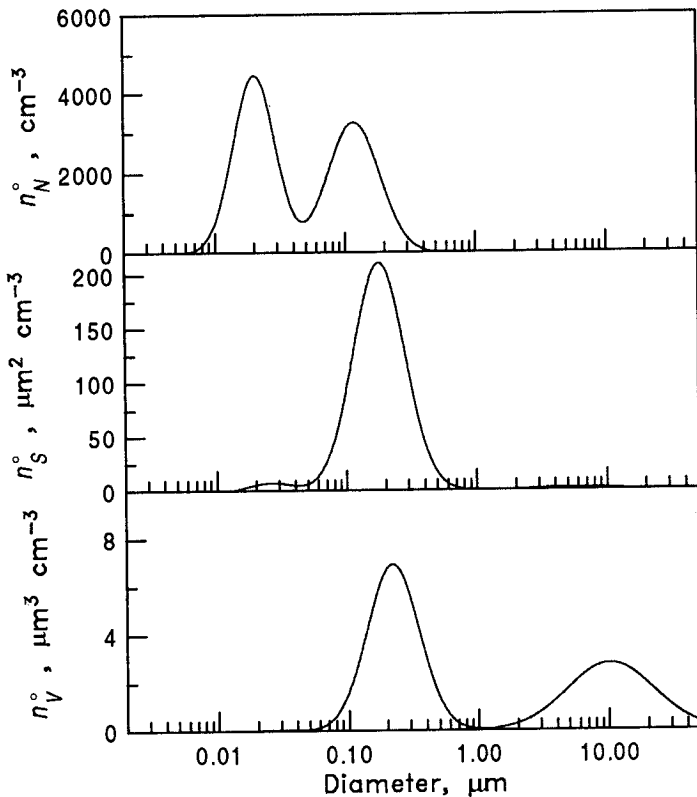


FIGURE 7.18 Typical remote continental aerosol number, surface and volume distributions.

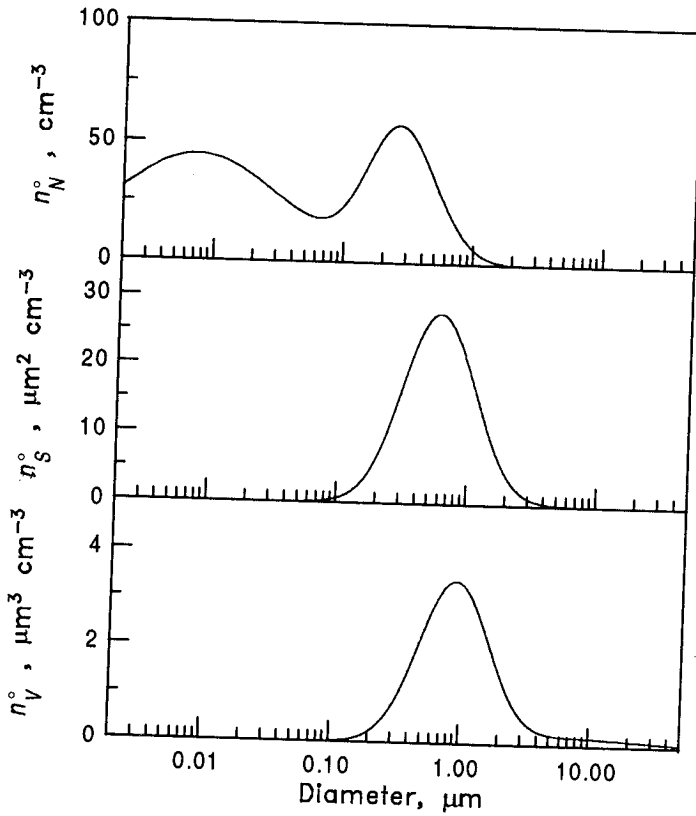


FIGURE 7.19 Typical free tropospheric aerosol number, surface, and volume distributions.

particles in the accumulation mode relative to lower tropospheric spectra, suggesting precipitation scavenging and deposition of smaller and larger particles (Leitch and Isaac, 1991).

7.2.6 Polar Aerosols

Polar aerosols, found close to the surface in the Arctic and Antarctica, reflect their aged character; their concentrations are very low. Collections of data from aerosol measurements in the Arctic have been presented by a number of investigators (Rahn, 1981; Shaw, 1985; Heintzenberg, 1989; Ottar, 1989). The number distribution appears practically monodisperse (Ito and Iwai, 1981) with a mean diameter of approximately $0.15 \mu\text{m}$; two more modes at 0.75 and $8 \mu\text{m}$ (Shaw, 1986; Jaenicke et al., 1992) (Figure 7.20) dominate the mass distribution.

During the winter and early spring (February to April) the Arctic aerosol has been found to be influenced significantly by anthropogenic sources, and the phenomenon is commonly referred to as Arctic Haze (Barrie, 1986). During this period the aerosol number concen-

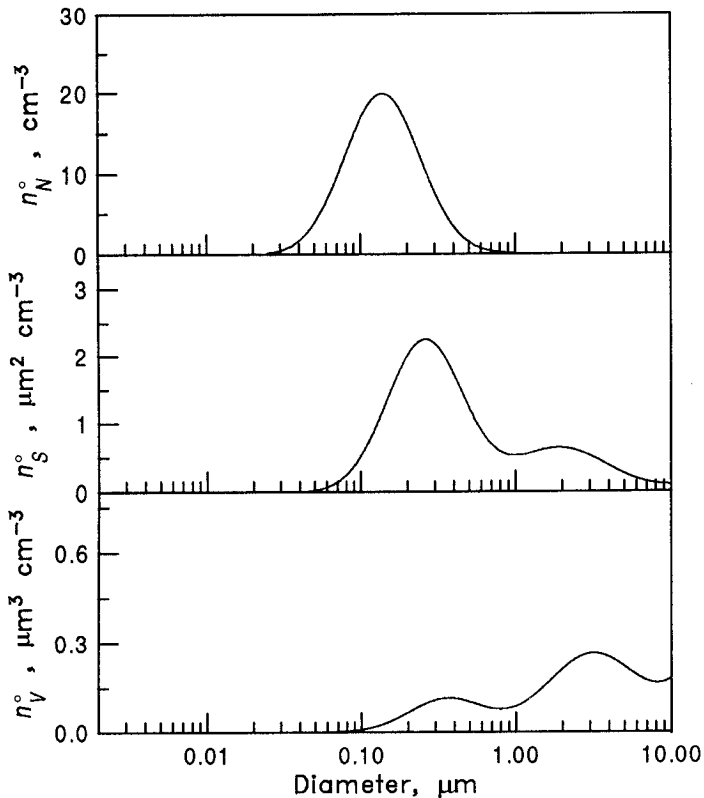


FIGURE 7.20 Typical polar aerosol number, surface, and volume distributions.

tration increases to over 200 cm^{-3} . The nucleation mode mean diameter is at $0.05 \mu\text{m}$ and the accumulation mode at $0.2 \mu\text{m}$ (Covert and Heintzenberg, 1993) (Figure 7.21). Similar measurements have been reported by Heintzenberg (1980), Radke et al. (1984), and (Shaw, 1984).

The polar aerosol contains carbonaceous material from midlatitude pollution sources, sulfate, sea salt from the surrounding ocean, and mineral dust from arid regions of the corresponding hemisphere. Aerosol PM_{10} concentrations in the polar regions are less than $5 \mu\text{g m}^{-3}$ with sulfate representing roughly 40% of the mass.

7.2.7 Desert Aerosols

Desert aerosol, of course present over deserts, actually extends considerably over adjacent regions such as oceans (Jaenicke and Schutz, 1978; d'Almeida and Schutz, 1983; Li et al., 1996). The shape of its size distribution is similar to that of remote continental aerosol but depends strongly on the wind velocity. Its number distribution tends to exhibit three overlapping modes at diameters of $0.01 \mu\text{m}$ or less, $0.05 \mu\text{m}$, and $10 \mu\text{m}$, respectively (Jaenicke, 1993) (Figure 7.22). An average composition of soils and crustal material is

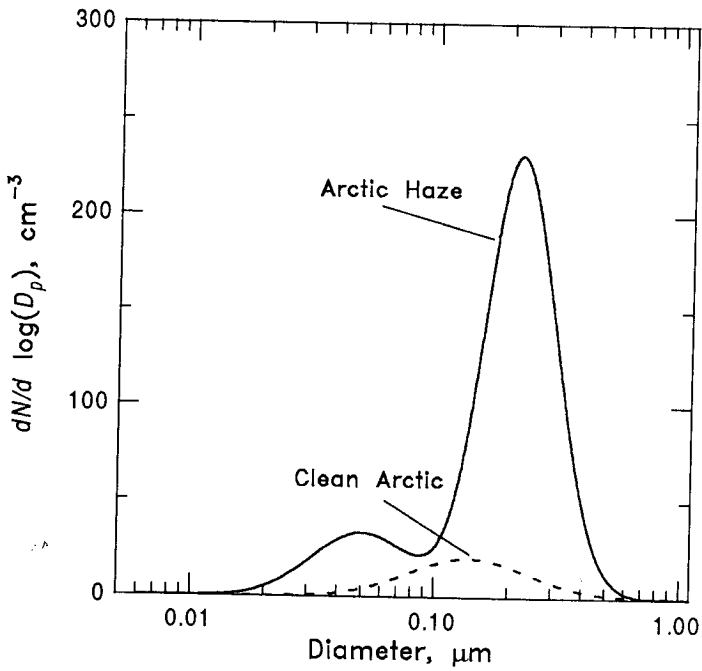


FIGURE 7.21 Comparison of the aerosol distribution during Arctic haze with the typical polar distribution.

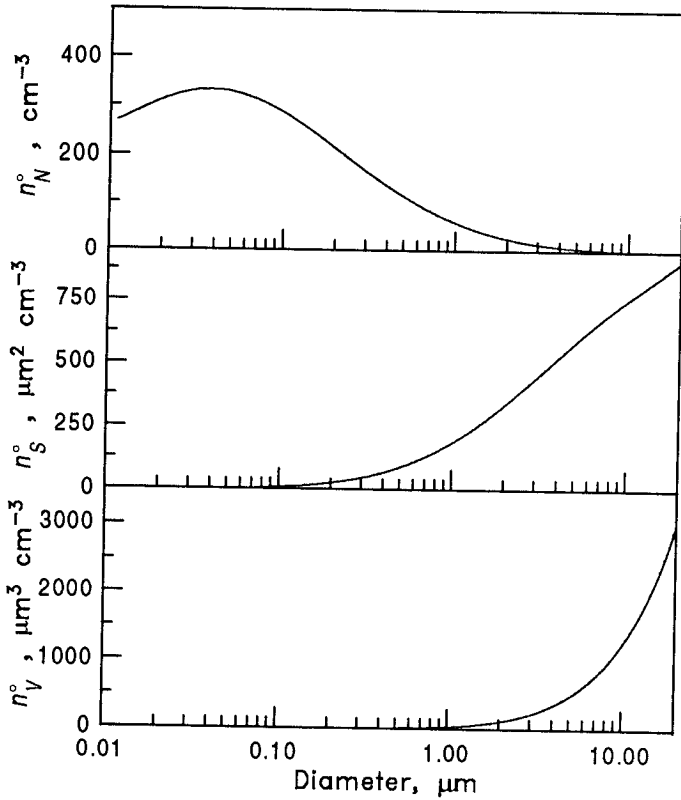


FIGURE 7.22 Typical desert aerosol number, surface, and volume distributions.

TABLE 7.4 Average Abundances of Major Elements in Soil and Crustal Rock

Element	Elemental Abundance (ppm by mass)	
	Soil	Crustal Rock
Si	330,000	311,000
Al	71,300	77,400
Fe	38,000	34,300
Ca	13,700	25,700
Mg	6,300	33,000
Na	6,300	31,900
K	13,600	29,500
Ti	4,600	4,400
Mn	850	670
Cr	200	48
V	100	98
Co	8	12

Source: Warneck (1988).

TABLE 7.5 Properties of Atmospheric Aerosol Types

Type	Number (cm ⁻³)	PM ₁ (μg m ⁻³)	PM ₁₀ (μg m ⁻³)
Urban (polluted)	10 ⁵ – 4 × 10 ⁶	30–150	100–300
Marine	100–400	1–4	10
Rural	2000–10,000	2.5–8	10–40
Remote continental	50–10,000	0.5–2.5	2–10

shown in Table 7.4. The soil composition is similar to that of the crustal rock, with the exception of the soluble elements such as Ca, Mg, and Na, that have lower relative concentrations in the soil.

Individual dust storms from the Sahara desert have been shown to transfer material from the northwest coast of Africa, across the Atlantic, to the east coast of the United States (Ott et al., 1991). For example, Prospero et al. (1987) suggested that enough Saharan dust is carried into the Miami area to significantly reduce visibility during the summer months. Similar dust transport occurs from the deserts of Asia across the Pacific Ocean (Prospero, 1995). While particles as large as 100 μm in diameter are found in the source regions, only particles smaller than 10 μm are transported over long distances, often further than 5000 km.

The average number and volume concentration of the major aerosol types are summarized in Table 7.5.

7.3 AEROSOL CHEMICAL COMPOSITION

Atmospheric aerosol particles contain sulfates, nitrates, ammonium, organic material, crustal species, sea salt, hydrogen ions, and water. From these species sulfate, ammonium, organic and elemental carbon, and certain transition metals are found predominantly in the

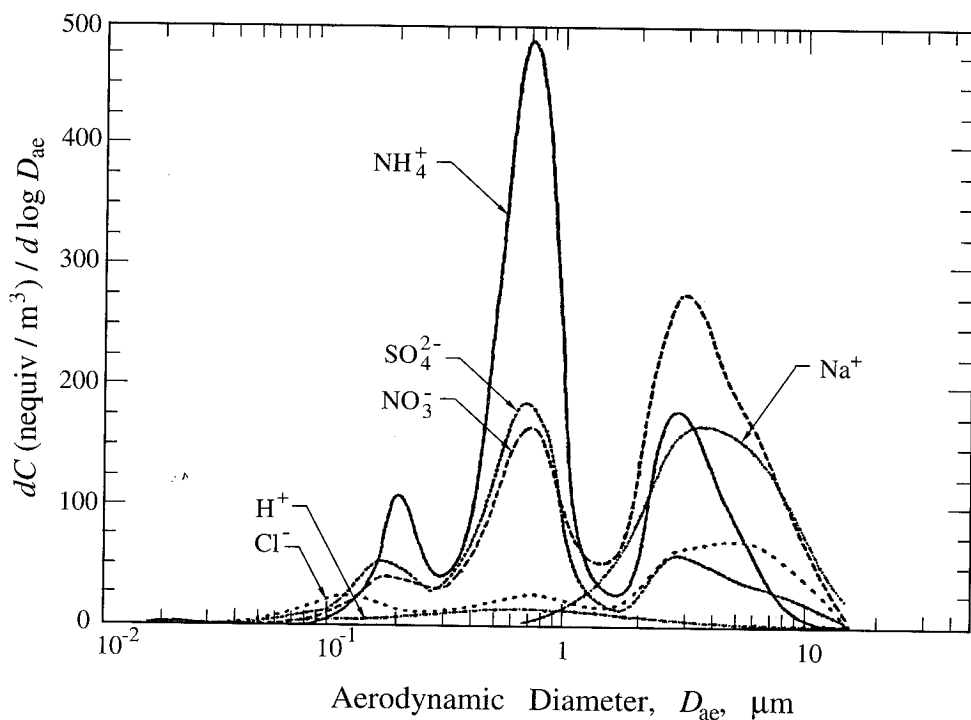


FIGURE 7.23 Measured size distributions of aerosol sulfate, ammonium, chloride, sodium, and hydrogen ion in Claremont, CA (Wall et al., 1988).

fine particles. Crustal materials, including silicon, calcium, magnesium, aluminum, and iron, and biogenic organic particles (pollen, spores, plant fragments) are usually in the coarse aerosol fraction. Nitrate can be found in both the fine and coarse modes. Fine nitrate is usually the result of the nitric acid/ammonia reaction for the formation of ammonium nitrate, while coarse nitrate is the product of coarse particle/nitric acid reactions.

A typical urban aerosol size/composition distribution is shown in Figure 7.23 (Wall et al., 1988). These results indicate that sulfate, nitrate, and ammonium have two modes in the 0.1 to 1.0 μm size range (the condensation and droplet modes), and a third one over 1 μm (coarse mode) (Figure 7.24). The condensation mode has a peak around 0.2 μm and is the result of condensation of secondary aerosol components from the gas phase. The droplet mode peaks around 0.7 μm in diameter and its existence is attributed to heterogeneous, aqueous-phase reactions discussed in Chapter 6 (Meng and Seinfeld, 1994). More than half of the nitrate is found in the coarse mode together with most of the sodium and chloride. This coarse nitrate is the result of reactions of nitric acid with sodium chloride or aerosol crustal material (see Chapter 9). This is an interesting case where secondary aerosol matter (nitrate) is formed through the reaction of a naturally produced material (sea salt or dust) and an anthropogenic pollutant (nitric acid).

More than 40 trace elements are routinely found in atmospheric particulate matter samples. These elements arise from dozens of different sources including combustion of coal, oil, wood burning, steel furnaces, boilers, smelters, dust, waste incineration, and break wear. Depending on their sources, these elements can be found in either the fine or the

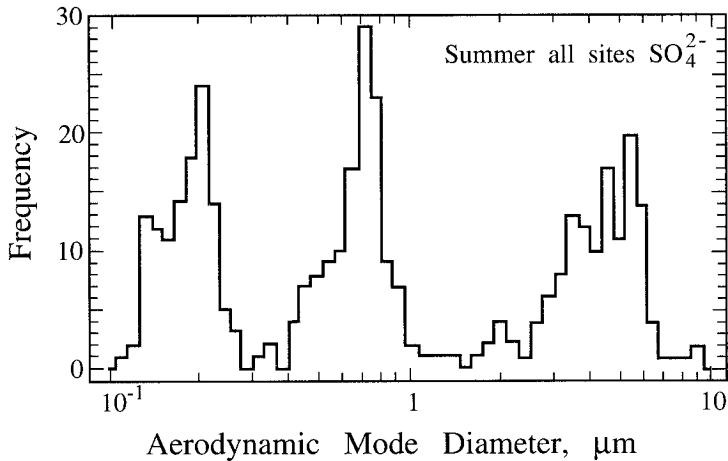


FIGURE 7.24 Frequency of observed occurrence of sulfate modes of various sizes as a function of mode diameter for Los Angeles during the summer of 1987 (John et al., 1990).

coarse mode. Concentrations of selected elements together with the size mode where these elements are usually found are shown in Table 7.6. The concentrations of these elements even for similar pollution levels vary over almost three orders of magnitude, indicating the strong effect of local sources. In general, elements such as lead, iron, and copper have the highest concentrations, while elements such as cobalt, mercury, and antimony are characterized by low concentrations. Elements produced during combustion usually exist in the

TABLE 7.6 Concentrations (ng m^{-3}) and Size Distribution of Various Elements Found in Atmospheric Particles

Element	Mode ^a	Concentration (ng m^{-3})		
		Remote	Rural	Urban
Fe	F and C	0.6–4,200	55–14,500	130–13,800
Pb	F	0.01–65	2–1,700	30–90,000
Zn	F	0.03–450	10–400	15–8,000
Cd	F	0.01–1	0.4–1,000	0.2–7,000
As	F	0.01–2	1–28	2–2,500
V	F and C	0.01–15	3–100	1–1,500
Cu	F and C	0.03–15	3–300	3–5,000
Mn	F and C	0.01–15	4–100	4–500
Hg	—	0.01–1	0.05–160	1–500
Ni	F and C	0.01–60	1–80	1–300
Sb	F	0–1	0.5–7	0.5–150
Cr	F and C	0.01–10	1–50	2–150
Co	F and C	0–1	0.1–10	0.2–100
Se	F and C	0.01–0.2	0.01–30	0.2–30

^aF = fine mode; C = coarse mode.

Source: Schroeder et al. (1987).

TABLE 7.7 Comparison of Ambient Fine and Coarse Particles

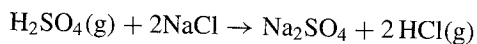
	Fine Particles	Coarse Particles
<i>Formation pathways</i>	Chemical reactions Nucleation Condensation Coagulation Cloud/fog processing	Mechanical disruption Suspension of dusts
<i>Composition</i>	Sulfate Nitrate Ammonium Hydrogen ion Elemental carbon (EC) Organic compounds Water Metals (Pb, Cd, V, Ni, Cu, Zn, Mn, Fe, etc.)	Resuspended dust Coal and oil fly ash Crustal element (Si, Al, Ti, Fe) oxides CaCO ₃ , NaCl Pollen, mold, spores Plant, animal debris Tire wear debris
<i>Solubility</i>	Largely soluble, hygroscopic	Largely insoluble and non-hygroscopic
<i>Sources</i>	Combustion (coal, oil, gasoline, diesel, wood) Gas-to-particle conversion of NO _x , SO ₃ , and VOCs Smelters, mills, etc.	Resuspension of industrial dust and soil Suspension of soil (farming, mining, unpaved roads) Biological sources Construction/demolition Ocean spray
<i>Atmospheric lifetime</i>	Days to weeks	Minutes to days
<i>Travel distance</i>	100s to 1000s of km	< to 10s of km

Source: Adapted from Wilson and Shuh (1997) and U.S. EPA (1996).

form of oxides (e.g., Fe₂O₃, Fe₃O₄, Al₂O₃), but their chemical form is in general uncertain.

A summary of chemical information regarding the coarse and fine modes is presented in Table 7.7.

The composition of sea salt reflects the composition of seawater enriched in organic material (marine-derived sterols, fatty alcohols, and fatty acids) that exists in the surface layer of the oceans (Schneider and Gagosian, 1985). Seawater contains 3.5% by weight sea salt and when first emitted the sea salt composition is the same as that of seawater (Table 7.8). Reactions on sea salt particles modify its chemical composition; for example, sodium chloride reacts with sulfuric acid vapor to produce sodium sulfate and hydrochloric acid vapor



leading to an apparent "chloride deficit" in the marine aerosol.

TABLE 7.8 Composition of Sea-Salt^a

Species	Percent by Weight
Cl	55.04
Na	30.61
SO ₄ ²⁻	7.68
Mg	3.69
Ca	1.16
K	1.1
Br	0.19
C (noncarbonate)	$3.5 \times 10^{-3} - 8.7 \times 10^{-3}$
Al	$4.6 \times 10^{-4} - 5.5 \times 10^{-3}$
Ba	1.4×10^{-4}
I	1.4×10^{-4}
Si	$1.4 \times 10^{-4} - 9.4 \times 10^{-3}$
NO ₃ ⁻	$3 \times 10^{-6} - 2 \times 10^{-3}$
Fe	$5 \times 10^{-5} - 5 \times 10^{-4}$
Zn	$1.4 \times 10^{-5} - 4 \times 10^{-5}$
Pb	$1.2 \times 10^{-5} - 1.4 \times 10^{-5}$
NH ₄ ⁺	$1.4 \times 10^{-6} - 1.4 \times 10^{-5}$
Mn	$2.5 \times 10^{-6} - 2.5 \times 10^{-5}$
V	9×10^{-7}

^aBased on the composition of seawater and ignoring atmospheric transformations.

7.4 VERTICAL VARIATION

The vertical distribution of aerosol mass concentration typically shows an exponential decrease with altitude up to a height H_p and a rather constant profile above that altitude (Gras, 1991). The aerosol mass concentration as a function of height can then be expressed as

$$M(z) = M(0) \exp\left(-\frac{z}{H_p}\right) \quad (7.59)$$

where $M(0)$ is the surface concentration and H_p the scale height. Jaenicke (1993) proposed values of H_p equal to 900 m for the marine, 730 m for the remote continental, 2000 m for the desert, and 30,000 m for the polar aerosol types. The corresponding vertical aerosol mass concentration profiles are shown in Figure 7.25.

The aerosol number concentration may increase or decrease exponentially with altitude and one suggestion of a form of the profile is (Jaenicke, 1993)

$$N(z) = N(0) \left[\exp\left(\frac{-z}{|H'_p|}\right) + \left(\frac{N_B}{N(0)}\right)^n \right]^n \quad (7.60)$$

where

$$n = \frac{H'_p}{|H'_p|} \quad (7.61)$$

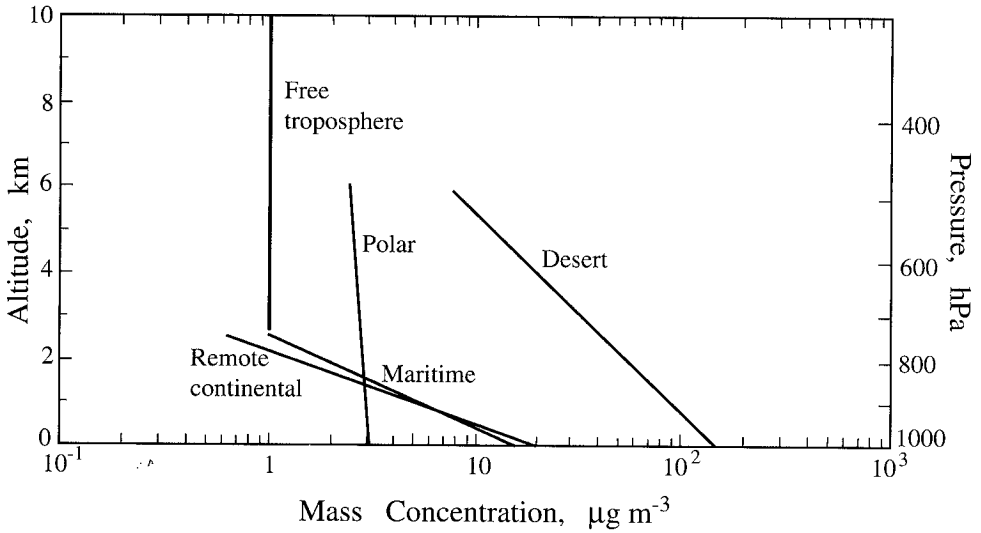


FIGURE 7.25 Representative vertical distribution of aerosol mass concentration (Jaenicke, 1993).

and N_B is the number concentration of the background aerosol aloft. For marine aerosol H'_p varies from -290 to 440 m. Note that if H'_p is negative $n = -1$, and (7.60) can be rewritten as

$$N(z) = N(0) \left[\exp\left(\frac{-z}{|H'_p|}\right) + \left(\frac{N(0)}{N_B}\right) \right]^{-1} \quad (7.62)$$

Because in this case $N(0) \ll N_B$, the equation has the correct limiting behavior both for $z \rightarrow 0$ and $z \rightarrow \infty$. Model vertical number concentration profiles are shown in Figure 7.26.

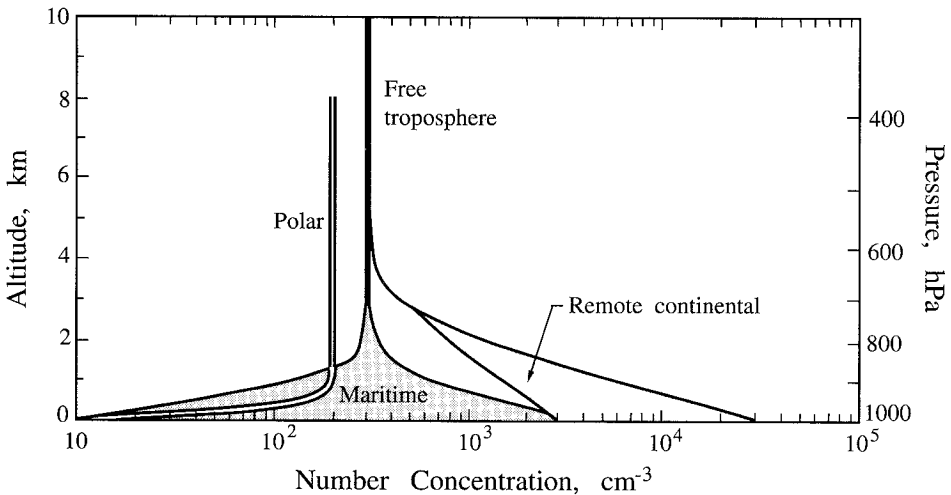


FIGURE 7.26 Representative vertical distribution of aerosol number concentration (Jaenicke, 1993). A range of concentrations is shown for marine and remote continental aerosols.

These vertical profiles are rough representations of long-term averages. Significant variability is observed in aerosol concentrations in anthropogenic plumes, areas influenced by local sources, or during nucleation events in the free troposphere.

REFERENCES

- Aitchison, J., and Brown, J. A. C. (1957) *The Lognormal Distribution Function*, Cambridge University Press, Cambridge.
- Barrie, L. A. (1986) Arctic air pollution: an overview of current knowledge, *Atmos. Environ.*, **20**, 643–663.
- Bashurova, V. S., et al. (1992) Measurements of atmospheric condensation nuclei size distributions in Siberia, *J. Aerosol Sci.* **23**, 191–199.
- Blanchard, D. C., and Cipriano, R. J. (1987) Biological regulation of climate, *Nature*, **330**, 526.
- Blanchard, D. C., and Woodcock, A. H. (1957) Bubble formation and modification in the sea and its meteorological significance, *Tellus*, **9**, 145–152.
- Covert, D. S., and Heintzenberg, J. (1993) Size distributions and chemical properties of aerosol at NY Ålesund, Svalbard, *Atmos. Environ.*, **27A**, 2989–2997.
- d'Almeida, G. A., and Schutz, L. (1983) Number, mass and volume distributions of mineral aerosol and soils of the Sahara, *J. Climate Appl. Meteorol.*, **22**, 233–243.
- Deepak, A., and Gali, G. (1991) *The International Global Aerosol Program (IGAP) Plan*. Deepak Publishing, Hampton, VA.
- Deirmendjian, D. (1969) *Electromagnetic Scattering on Spherical Polydispersions*, Elsevier, New York.
- De Leeuw, G. (1986) Vertical profiles of giant particles close above the sea surface, *Tellus*, **38B**, 51–61.
- Fitzgerald, J. W. (1991) Marine aerosols: a review, *Atmos. Environ.*, **25A**, 533–545.
- Gras, J. L. (1991) Southern hemisphere tropospheric aerosol microphysics, *J. Geophys. Res.*, **96**, 5345–5356.
- Haaf, W., and Jaenicke, R. (1980) Results of improved size distribution measurements in the Aitken range of atmospheric aerosols, *J. Aerosol Sci.*, **11**, 321–330.
- Heintzenberg, J. (1980) Particle size distribution and optical properties of Arctic haze, *Tellus*, **32**, 251–260.
- Heintzenberg, J. (1989) Arctic haze: air pollution in polar regions, *Ambio*, **18**, 50–55.
- Hering, S. V., and Friedlander, S. K. (1982) Origins of aerosol sulfur size distributions in the Los Angeles basin, *Atmos. Environ.*, **16**, 2647–2656.
- Hinds, W. C. (1982) *Aerosol Technology*, Wiley, New York.
- Hobbs, P. V., Bowdle, D. A., and Radke, L. F. (1985) Particles in the lower troposphere over the high plains of the United States. 1. Size distributions, elemental compositions, and morphologies, *J. Climate Appl. Meteorol.*, **24**, 1344–1356.
- Hoppel, W. A., Fitzgerald, J. W., Frick, G. M., Larson, R. E., and Mack, E. J. (1989) *Atmospheric Aerosol Size Distributions and Optical Properties in the Marine Boundary Layer over the Atlantic Ocean*. NRL Report 9188, Washington, DC.
- Ito, T., and Iwai, K. (1981) On the sudden increase in the concentration of aitken particles in the Antarctic atmosphere, *J. Meteorol. Soc. Jpn.*, **59**, 262–271.
- Jaenicke, R. (1993) Tropospheric aerosols, in *Aerosol–Cloud–Climate Interactions*, edited by P. V. Hobbs. Academic Press, San Diego, CA; pp. 1–31.
- Jaenicke, R., and Schutz, L. (1978) Comprehensive study of physical and chemical properties of the surface aerosol in the Cape Verde Islands region, *J. Geophys. Res.*, **83**, 3583–3599.

- Jaenicke R., Dreiling V., Lehmann E., Koutsenogii, P. K., and Stingl, J. (1992) Condensation nuclei at the German Antarctic Station Vonneymayer, *Tellus*, **44B**, 311–317
- John, W., Wall, S. M., Ondo, J. L., and Winklmayr, W. (1990) Modes in the size distributions of atmospheric inorganic aerosol, *Atmos. Environ.*, **24A**, 2349–2359.
- Koutsenogii, P. K., and Jaenicke, R. (1994) Number concentration and size distribution of atmospheric aerosol in Siberia, *J. Aerosol Sci.*, **25**, 377–383.
- Koutsenogii, P. K., Bufetov, N. S., and Drosdova, V. I. (1993) Ion composition of atmospheric aerosol near Lake Baikal, *Atmos. Environ.*, **27A**, 1629–1633.
- Leaitch, W. R., and Isaac, G. A. (1991) Tropospheric aerosol size distributions from 1982 to 1988 over Eastern North America, *Atmos. Environ.*, **25A**, 601–619.
- Li, X., Maring, H., Savoie, D., Voss, K., and Prospero, J. M. (1996) Dominance of mineral dust in aerosol light scattering in the North Atlantic trade winds, *Nature*, **380**, 416–419.
- Meng, Z., and Seinfeld, J. H. (1994) On the source of the submicrometer droplet mode of urban and regional aerosols, *Aerosol Sci. Technol.*, **20**, 253–265.
- Mészáros, A., and Vissy, K. (1974) Concentration, size distribution and chemical nature of atmospheric aerosol particles in remote ocean areas, *J. Aerosol Sci.*, **5**, 101–109.
- Monahan, E. C., Fairall, C. W., Davidson, K. L., and Jones-Boyle, P. (1983) Observed inter-relationships amongst 10-m-elevation winds, oceanic whitecaps, and marine aerosols, *Q. J. R. Meteorol. Soc.*, **109**, 379–392.
- O'Dowd, C. D., and Smith, M. H. (1993) Physicochemical properties of aerosols over the Northeast Atlantic: evidence for wind-speed related submicron sea-salt aerosol production, *J. Geophys. Res.*, **98**, 1137–1149.
- Ott, S. T., Ott, A., Martin, D. W., and Young, J. A. (1991) Analysis of trans-Atlantic saharan dust outbreak based on satellite and GATE data, *Mon. Weather Rev.*, **119**, 1832–1850.
- Ottar, B. (1989) Arctic air pollution: a Norwegian perspective, *Atmos. Environ.*, **23**, 2349–2356.
- Prospero, J. M. (1995) The atmospheric transport of particles to the ocean, in SCOPE Report: Particle Flux in the Ocean. Ittekkot, V., Honjo, S., Depetris, P. J. (Eds.), Wiley, New York, NY.
- Prospero, J. M., Nees, R.T., and Uematsu, M. (1987) Deposition rate of particulate and dissolved aluminum derived from sahara dust in precipitation in Miami, Florida, *J. Geophys. Res.*, **92**, 14723–14731.
- Pruppacher, H. R., and Klett, J. D. (1980) *Microphysics of Cloud and Precipitation*, D. Reidel, Dordrecht, The Netherlands.
- Radke, L. F., Lyons, J. H., Hegg, D. A., and Hobbs, P. V. (1984) Airborne observations of Arctic aerosols. I: Characteristics of Arctic haze, *Geophys. Res. Lett.*, **11**, 369–372.
- Rahn, K. (1981) Relative importance of North America and Eurasia as sources of Arctic aerosol, *Atmos. Environ.*, **15**, 1447–1456.
- Savoie, D. L., and Prospero, J. M. (1989) Comparison of oceanic and continental sources of non-sea-salt sulfate over the Pacific ocean, *Nature*, **339**, 685–687.
- Schneider, J. K., and Gagosian, R. B. (1985) Particle size distribution of lipids in aerosols off the coast of Peru, *J. Geophys. Res.*, **90**, 7889–7898.
- Schroeder, W. H., Dobson, M., Kane, D. M., and Johnson, N. D. (1987) Toxic trace elements associated with airborne particulate matter: a review, *J. Air Pollut. Cont. Assoc.*, **37**, 1267–1285.
- Shaw, G. E. (1984) Microparticle size spectrum of Arctic haze, *Geophys. Res. Lett.*, **11**, 409–412.
- Shaw, G. E. (1985) Aerosol measurements in Central Alaska 1982–1984, *Atmos. Environ.*, **19**, 2025–2031.
- Shaw, G. E. (1986) On physical properties of aerosols at Ross Island, Antarctica, *J. Aerosol Sci.*, **17**, 937–945.

- United States Environmental Protection Agency (1996) Air Quality Criteria for Particulate Matter, EPA/600/P-95/001, Research Triangle Park, NC.
- Wall, S. M., John, W., and Ondo, J. L. (1988) Measurement of aerosol size distributions for nitrate and major ionic species, *Atmos. Environ.*, **22**, 1649–1656.
- Warneck, P. (1988) *Chemistry of the Natural Atmosphere*, Academic Press, San Diego, CA.
- Whitby, K. T., and Sverdrup, G. M. (1980) California aerosols: their physical and chemical characteristics, in *The Character and Origins of Atmospheric Aerosols: A Digest of Results from the California Aerosol Characterization Experiment (ACHEX)*, Hidy, G. M., Mueller, P. K., Grosjean, D., Appel, B. R., and Wesolowski, J. J. (Eds.), pp. 477–517. *Advances in Environmental Science and Technology*, Vol. **9**, Wiley, New York, NY.
- Wilson, W. E., and Shuh, H. H. (1997) Fine and coarse particles: concentration relationships relevant to epidemiological studies, *J. Air Waste Manage. Assoc.*, (in press).

PROBLEMS

- 7.1_A Given the following data on the number of aerosol particles in the size ranges listed, tabulate and plot the normalized size distributions $\bar{n}_N(D_p) = n_N(D_p)/N$ and $\bar{n}_N^\circ(\log D_p) = n_N^\circ(\log D_p)/N$ as discrete histograms.

Size Interval (μm)	Mean of Size Interval (μm)	Number of Particles in Interval
0–0.2	0.1	10
0.2–0.4	0.3	80
0.4–0.6	0.5	132
0.6–0.8	0.7	142
0.8–1.0	0.9	138
1.0–1.2	1.1	112
1.2–1.4	1.3	75
1.4–1.6	1.5	65
1.6–1.8	1.7	52
1.8–2.1	1.95	65
2.1–2.7	2.4	62
2.7–3.6	3.15	32
3.6–5.1	4.35	35

- 7.2_A For the data given in Problem 7.1, plot the surface area and volume distributions $n_S(D_p)$, $n_S^\circ(\log D_p)$, $n_V(D_p)$ and $n_V^\circ(\log D_p)$ in both non-normalized and normalized form as discrete histograms.
- 7.3_A You are given an aerosol size distribution function $n_M(m)$ such that $n_M(m) dm =$ aerosol mass per cm^3 of air contained in particles having masses in the range m to $m + dm$. It is desired to convert that distribution function to a mass distribution based on $\log D_p$. Show that

$$n_M^\circ(\log D_p) = 6.9 m n_M(m)$$

7.4_B Show that the variance of the size distribution of a log-normally distributed aerosol is

$$\overline{D}_p^2 [\exp(\ln^2 \sigma_g) - 1]$$

7.5_A Starting with semilogarithmic graph paper, construct a log-probability coordinate axis and show that a log-normal distribution plots as a straight line on these coordinates.

7.6_A The data given below were obtained for a log-normally distributed aerosol size distribution:

Size Interval (μm)	Geometric Mean of Size Interval (μm)	Number of Particles in Interval ^a
0.1–0.2	0.1414	50
0.2–0.4	0.2828	460
0.4–0.7	0.5292	1055
0.7–1.0	0.8367	980
1.0–2.0	1.414	1705
2.0–4.0	2.828	680
4.0–7.0	5.292	102
7.0–10	8.367	10
10–20	14.14	2

^aAssume that the particles are spheres with density $\rho = 1.5 \text{ g cm}^{-3}$.

a. Complete the above table by computing the following quantities: $\Delta N_i / \Delta D_{pi}$, $\Delta N_i / N \Delta D_{pi}$, $\Delta S_i / \Delta D_{pi}$, $\Delta S_i / S \Delta D_{pi}$, $\Delta M_i / \Delta D_{pi}$, $\Delta M_i / M \Delta D_{pi}$, $\Delta N_i / \Delta \log D_{pi}$, $\Delta N_i / N \Delta \log D_{pi}$, $\Delta S_i / \Delta \log D_{pi}$, $\Delta S_i / S \Delta \log D_{pi}$, and $\Delta M_i / \Delta \log D_{pi}$, $\Delta M_i / M \Delta \log D_{pi}$, where M = particle mass.

b. Plot $\Delta N_i / \Delta \log D_{pi}$, $\Delta S_i / \Delta \log D_{pi}$, and $\Delta M_i / \Delta \log D_{pi}$ as histograms.

c. Determine the geometric mean diameter and geometric standard deviation of the log-normal distribution to which these data adhere and plot the continuous distributions on the three plots from part (b).

7.7_B For a log-normally distributed aerosol different mean diameters can be defined by

$$\overline{D}_{pv} = \overline{D}_{pg} \exp(\nu \ln^2 \sigma_g)$$

where ν is a parameter that defines the particular mean diameter of interest. Show that

Diameter	ν
Mode (most frequent value)	-1
Geometric mean or median	0
Number (arithmetic) mean	0.5
Surface area mean	1
Mass mean	1.5
Surface area median	2
Volume median	3

Plot a normalized log-normal particle size distribution over a range of D_p from 0 to $7 \mu\text{m}$ with $\bar{D}_{pg} = 1.0 \mu\text{m}$ and $\sigma_g = 2.0$ and identify each of the above diameters on the plot. *Hint:* You may find this integral of use:

$$\int_{L_1}^{L_2} e^{ru} \exp\left(-\frac{(u - \bar{u})^2}{2\sigma_u^2}\right) du$$

$$= (\pi/2)^{1/2} \sigma_u e^{r\bar{u}} e^{r^2\sigma_u^2/2} \left[\operatorname{erf}\left(\frac{L_2 - (\bar{u} + r\sigma_u^2)}{\sqrt{2}\sigma_u}\right) - \operatorname{erf}\left(\frac{L_1 - (\bar{u} + r\sigma_u^2)}{\sqrt{2}\sigma_u}\right) \right]$$

7.8_B Assume that an aerosol has a log-normal distribution with $\bar{D}_{pg} = 5.5 \mu\text{m}$ and $\sigma_g = 1.36$.

- a. Plot the number and volume distributions of this aerosol on log-probability paper.
- b. It is desired to represent this aerosol by a distribution of the form

$$F_V(D_p) = 1 - \exp(-cD_p^b)$$

where $F_V(D_p)$ is the fraction of the total aerosol volume in particles of diameter less than D_p . Determine the values of the constants c and b needed to match this distribution to the given aerosol.

7.9_B Given the following size frequency for a dust:

Size Interval (μm)	% by Number
7–17.5	10
17.5–21	10
21–25	10
25–28	10
28–30	10
30–33	10
33–36	10
36–41	10
41–49	10
49–70	10

- a. Plot the cumulative frequency distributions (in %) of the number, surface area, and mass on linear graph paper assuming all particles are spheres with $\rho_p = 1.6 \text{ g cm}^{-3}$.
- b. Is this a log-normally distributed dust?

7.10_B The following particle size distribution data are available for an aerosol:

D_p	% by Volume Less Than
9.8	3.2
13.8	10.0
19.6	26.7
27.7	46.8
39.1	72.0
55.3	87.5

- What is the volume median diameter and geometric standard deviation of the volume distribution of this aerosol?
- What is the surface area median diameter?