GLOBAL CIRCULATION

CONTENTS

11 Solar radiation absorbed in the tropics exceeds infrared loss, causing heat to accumulate. The opposite is true at the poles, where there is net radiative cooling. Such **radiative differential heating** between poles and equator creates an imbalance in the atmosphere/ocean system (Fig 11.1a).

As expected by LeChatelier's Principle, this unstable system reacts to undo the imbalance (Fig 11.1b). Warm tropical air rises due to buoyancy and flows toward the poles, and the cold polar air sinks and flows toward the equator.

Because the radiative forcings are unremitting, a ceaseless movement of wind and ocean currents results in what we call the **general circulation** or **global circulation**. The circulation cannot keep up with the continued destabilization, causing the tropics to remain slightly warmer than the poles.

The atmosphere is said to be **baroclinic** where there is a horizontal temperature gradient. This gradient creates the jet stream at midlatitudes. Baroclinic instabilities cause the jet stream to meander and cause the eddy-like storm systems and waves that are associated with midlatitude weather.

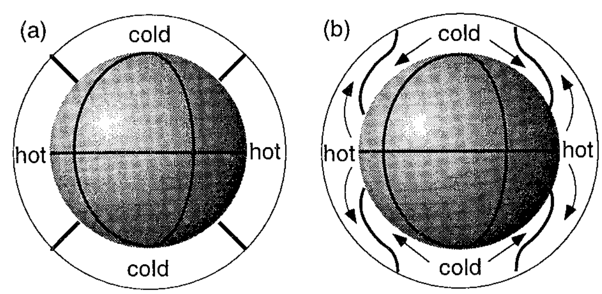


Figure 11.1
Earth/atmosphere system, showing that (a) differential heating by radiation causes (b) atmospheric circulations.

NOMENCLATURE

Latitude lines are **parallels**, and east-west winds are called **zonal** flow (Fig 11.2). Longitude lines are **meridians**, and north-south winds are called **meridional** flow.

Midlatitudes are the region between about 30° and 60° latitude. The **subtropical** zone is at roughly 30° latitude, and the **subpolar** zone is at 60° latitude, both of which partially overlap midlatitudes. **Tropics** span the equator, and **polar** regions are near the earth's poles. **Extratropical** refers to everything poleward from roughly 30°N and 30°S. Similar terminology is used in the southern hemisphere.

For example, extratropical cyclones are low-pressure centers, that are typically called lows and labeled with L, found in midlatitudes or high latitudes (> 60° N or S, roughly). Tropical cyclones include hurricanes and typhoons, and other lows in tropical regions.

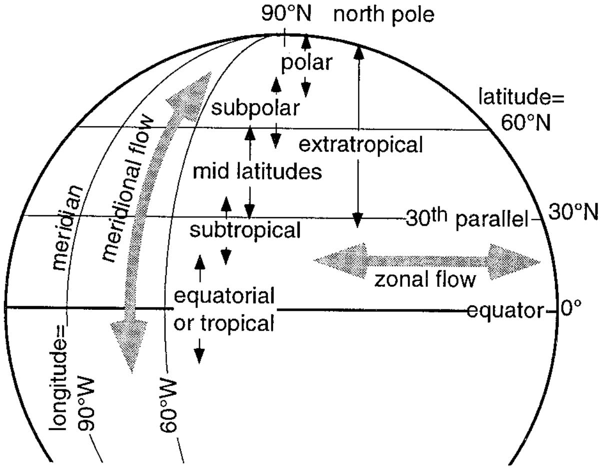


Figure 11.2
Global nomenclature.

DIFFERENTIAL HEATING

The balance between incoming and outgoing radiation drives the global circulation. Outgoing infrared radiation depends on the earth's temperature, as discussed in Chapter 2. Because the general circulation cannot completely eliminate the temperature disparity across the globe, there is a residual north-south temperature gradient that we will examine first.

Meridional Temperature Gradient

Sea-level temperatures are warmer near the equator than the poles, when averaged over a whole year and averaged along lines of constant latitude (Fig 11.3a). Although the Northern Hemisphere is slightly cooler than the Southern, an idealization of the meridional temperature variation at sea level is

$$T_{sea\ level} \approx a + b \cdot \left[\frac{3}{2} \cdot \left(\frac{2}{3} + \sin^2 \phi \right) \cdot \cos^3 \phi \right] \qquad (11.1)$$

where $a \cong -12°C$, $b \cong 40°C$, and ϕ is latitude. Parameter b represents a temperature difference between equator and pole, so it could also have been written as $b \cong 40$ K.

In the tropics there is little temperature variation – it is hot everywhere. However, in midlatitudes, there is a significant north-south temperature gradient that supports a variety of storm systems. Although eq. (11.1) might seem unnecessarily complex, it is designed to give sufficient uniformity in the tropics to support midlatitude jet streams.

The gradient associated with eq. (11.1) is

$$\frac{\Delta T}{\Delta y} \approx -b \cdot c \cdot \left[\sin^3 \phi \cdot \cos^2 \phi \right] \qquad (11.2)$$

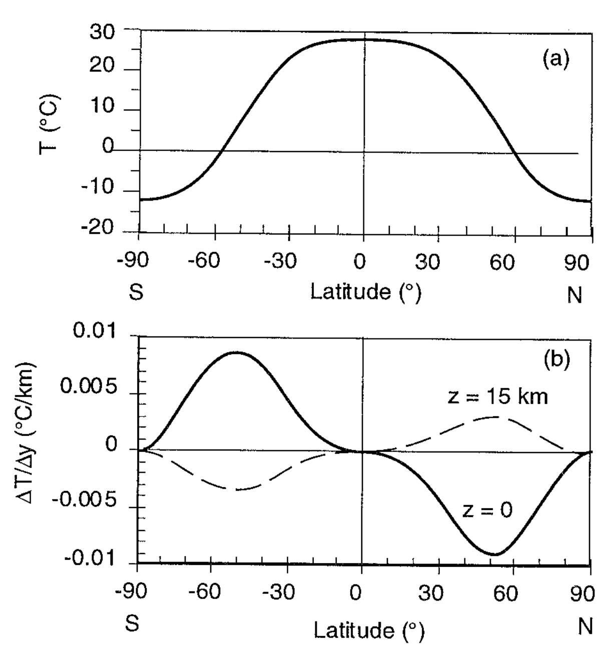


Figure 11.3
Annually and zonally averaged (a) temperature at sea level (idealized), and (b) north-south temperature gradient at sea level and at 15 km altitude.

where $c = 1.18\text{x}10^{-3}$ km^{-1}, and $b \cong 40°C$ at sea level, as before. The sea-level meridional temperature gradient is plotted in Fig 11.3b.

The magnitude of the north-south temperature gradient decreases with altitude, and changes sign in the stratosphere. Namely, in the stratosphere it is cold in the tropics and warmer in the polar regions. For this reason, parameter b in eqs. (11.1) and (11.2) can be approximated by

$$b \approx b_1 \cdot \left(1 - \frac{z}{z_T}\right) \qquad (11.3)$$

where $b_1 = 40°C$, z is height above sea level, and $z_T \cong 11$ km is the average depth of the troposphere. With this change to eq. (11.2), the horizontal temperature gradient at $z = 15$ km is also plotted in Fig 11.3b.

Solved Example

Find the meridional temperature and temperature gradient at 45°N latitude, at sea level.

Solution

Given: $\phi = 45°$

Find: $T = ?$ °C, $\Delta T/\Delta y = ?$ °C/km

Use eq. (11.1):

$$T_{sl} \approx -12°C + (40°C)\cdot\left[\frac{3}{2}\cdot\left(\frac{2}{3} + \sin^2 45°\right)\cdot\cos^3 45°\right]$$

= <u>12.75 °C</u>

Use eq. (11.2):

$$\frac{\Delta T}{\Delta y} \approx -(40°C)\cdot(1.18\times10^{-3})\cdot\left[\sin^3 45°\cdot\cos^2 45°\right]$$

= <u>–0.0083 °C/km</u>

Check: Units OK. Physics OK. Agrees with Figs.

Discussion: Temperature decreases toward the north in the northern hemisphere, which gives the negative sign for the gradient. As a quick check, from Fig 11.3a the temperature decreases by about 9°C between 40°N to 50°N latitude. But each 1° of latitude equals 111 km of distance, y. This temperature gradient of –9°C/(1110 km) agrees with the answer above.

BEYOND ALGEBRA • Temperature Gradient

Problem: Derive eq. (11.2) from eq. (11.1).

Solution: Given: $T \approx a + b\cdot\left[\frac{3}{2}\cdot\left(\frac{2}{3} + \sin^2\phi\right)\cdot\cos^3\phi\right]$

Find: $$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \phi}\cdot\frac{\partial \phi}{\partial y} \qquad (a)$$

The first factor on the RHS is: $\frac{\partial T}{\partial \phi} = b\cdot\left(\frac{3}{2}\right)\cdot$

$$\left[(2\sin\phi\cdot\cos\phi)\cos^3\phi - 3\left(\frac{2}{3} + \sin^2\phi\right)\cos^2\phi\cdot\sin\phi\right]$$

Taking $\sin\phi\cdot\cos^2\phi$ out of the square brackets:

$$\frac{\partial T}{\partial \phi} = b\left(\frac{3}{2}\right)\sin\phi\cdot\cos^2\phi\cdot\left[2\cos^2\phi - 2 - 3\sin^2\phi\right]$$

But $\cos^2\phi = 1 - \sin^2\phi$, thus $2\cos^2\phi = 2 - 2\sin^2\phi$:

$$\frac{\partial T}{\partial \phi} = b\left(\frac{3}{2}\right)\sin\phi\cdot\cos^2\phi\cdot\left[-5\sin^2\phi\right] \qquad (b)$$

Inserting eq. (b) into (a) gives:

$$\frac{\partial T}{\partial y} = -b\cdot\left(\frac{15}{2}\cdot\frac{\partial\phi}{\partial y}\right)\cdot\sin^3\phi\cdot\cos^2\phi \qquad (c)$$

Define: $\left(\frac{15}{2}\cdot\frac{\partial\phi}{\partial y}\right) \equiv c$ Thus, the final answer is:

$$\boxed{\frac{\partial T}{\partial y} = -b\cdot c\cdot\sin^3\phi\cdot\cos^2\phi} \qquad (11.2)$$

All that remains is to find c. Note that $\partial\phi/\partial y$ is the change of latitude per distance traveled north. The total change in latitude to circumnavigate the earth from pole to pole is 2π radians, and the circumference of the earth is $2\pi R$ where $R = 6356.766$ km is the average radius of the earth. Thus:

$$\frac{\partial\phi}{\partial y} = \frac{2\pi}{2\pi\cdot R} = \frac{1}{R}$$

The constant c is then:

$$c = \left(\frac{15}{2}\cdot\frac{1}{R}\right) = 1.18\times10^{-3}\text{km}^{-1} \qquad (e)$$

Check: The neighboring solved example verified that eq. (11.2) is a reasonable answer, given the temperature defined by eq. (11.1).

Caution: Eq. (11.1) is only a crude approximation to nature, designed to be convenient for analytical calculations of temperature gradients, thermal winds, IR emissions, etc. While it serves an education purpose here, more accurate models should be used elsewhere.

Radiative Forcings

Idealize the zonally- and annually-averaged outgoing radiative flux E_{out} by a truncated curve:

$$E_{out} \approx \varepsilon \cdot \sigma \cdot T^4_{sea\ level} \qquad (11.4a)$$

with

$$E_{out} \leq 260 \ \ \mathrm{W/m^2} \qquad (11.4b)$$

where $\sigma = 5.67\mathrm{x}10^{-8}$ W·m^{-2}·K^{-4} is the Stefan-Boltzmann constant, and $\varepsilon \cong 0.7$ is an effective emissivity that crudely compensates for emission from altitudes above the surface. The reason for truncating the top of the curve is because extensive cloudiness in the tropics limits the emissions there.

Incoming radiative flux E_{in}, averaged zonally and annually, is idealized as

$$|E_{in}| \approx |E_1| \cdot \cos(\phi) \qquad (11.5a)$$

with

$$|E_{in}| \leq 310 \ \ \mathrm{W/m^2} \qquad (11.5b)$$

where $|E_1| \cong 315$ W/m^2.

Solved Example(§)

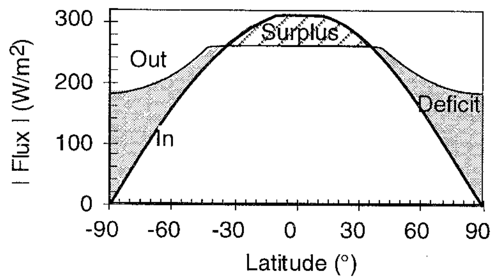
Plot zonally-averaged incoming and outgoing radiation vs. latitude, for each 5° of latitude.

Solution

Find: E_{in} & E_{out} (W/m^2) vs. ϕ (°)

Use eqs. (11.1), (11.4), and (11.5) in a spreadsheet, a portion of which is shown below:

ϕ (°)	ϕ (rad)	E_{in} (W/m^2)	T_{sfc} (°C)	E_{out} (W/m^2)
90	1.571	0.0	-12.0	184.2
85	1.484	27.5	-11.9	184.4
80	1.396	54.7	-11.5	185.6
75	1.309	81.5	-10.3	188.9
etc.				



Check: Units OK. Physics OK.

Discussion: In tropical regions where incoming radiation exceeds outgoing, there is a surplus of energy. Similarly, near the poles there is a deficit. This is the "differential heating" between the equator and poles.

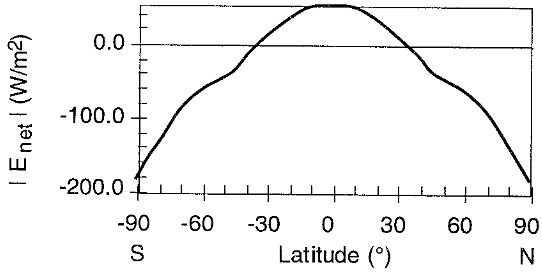


Figure 11.4
Magnitude of net radiative flux into the troposphere.

The net radiative heat flux per vertical column of atmosphere is simply

$$E_{net} = |E_{in}| - E_{out} \qquad (11.6)$$

which is plotted in Fig 11.4.

Eq. (11.6) and Fig 11.4 can be deceiving, because latitude belts have shorter circumference near the poles than near the equator. Namely, there are fewer square meters near the poles that experience the net deficit than there are near the equator that experience the net surplus.

To compensate for the shrinking latitude belts, multiply the radiative flux by the circumference of the belt $2\pi \cdot R_{earth} \cdot \cos(\phi)$, to give a more appropriate measure of heating vs. latitude:

$$E_{\phi} = 2\pi \cdot R_{earth} \cdot \cos(\phi) \cdot E \qquad (11.7)$$

where E is the magnitude of incoming or outgoing radiative flux, and R_{earth} = 6357 km is the average earth radius. These E_{ϕ} values are plotted in Fig 11.5.

The difference D_{ϕ} between incoming and outgoing values of E_{ϕ} is plotted in Fig 11.6.

$$D_{\phi} = E_{\phi\ in} - E_{\phi\ out} \qquad (11.8)$$

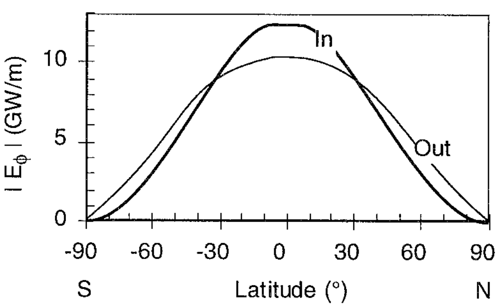


Figure 11.5
Magnitude of radiation accumulated around latitude belts.

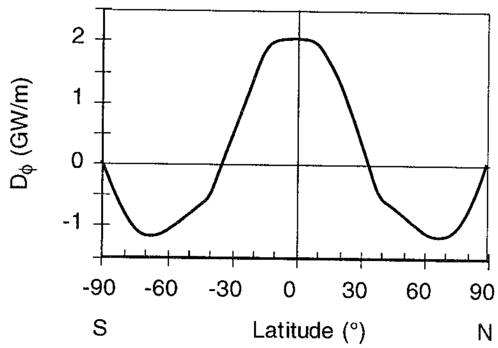


Figure 11.6
Difference between incoming and outgoing radiation, accumulated around latitude belts. This is the "differential heating."

To interpret this curve, picture a sidewalk built around the world along a latitude line. If this sidewalk is 1 m wide, then Fig 11.6 gives the number of gigawatts of net radiative power absorbed by the sidewalk. This curve shows the radiative **differential heating**. The areas under the positive and negative portions of the curve in Fig 11.6 balance, leaving the earth in overall equilibrium.

ON DOING SCIENCE • Toy Models

Some problems in meteorology are so complex, and involve so many interacting variables and processes, that they are intimidating if not impossible to solve. However, it is sometimes possible to gain insight into fundamental aspects of the problem by approximating the true physics by idealized, simplified physics. Such an approximation is sometimes called a **toy model**.

The meridional variation of temperature given by eq. (11.1) is an example of a toy model. It was designed to capture only the dominant temperature variations in a way that could be used to analytically calculate temperature gradients, energy balances and geostrophic winds. It is a model of the atmosphere — a toy model, not a complete model.

Toy models are used extensively to study climate change. For example, the greenhouse effect is examined using toy models in the last chapter of this book. Toy models capture only the dominant effects, and neglect the subtleties. They should never be used to infer the details of a process, particularly in situations where two large but opposite processes nearly cancel each other.

For other examples of toy models applied to the environment, see John Harte's 1988 book *Consider a Spherical Cow*, University Science Books. 283 pp.

Heat Transport Needed

The radiative imbalance in Fig 11.6 must be compensated by atmospheric and oceanic circulations. The net meridional transport is zero at the poles by definition. Starting at the north pole, and summing over all the "sidewalks" down to any latitude Φ of interest gives the total atmospheric and oceanic northward transport Tr needed to compensate the radiation:

$$Tr = \sum_{\phi=90°}^{\Phi} (-D_\phi) \cdot \Delta y \tag{11.9}$$

where Δy is the width of the sidewalk.

Winds transport some of this heat. These winds are generated by the horizontal temperature gradient, as described next.

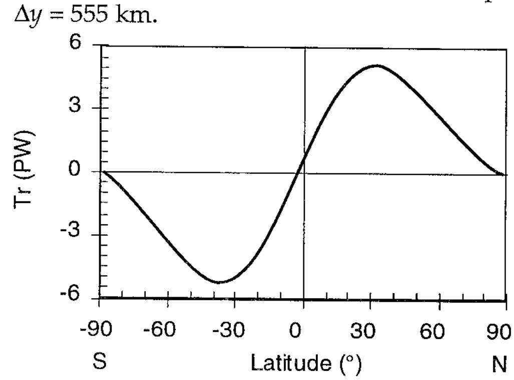
Solved Example(§)

Plot the net atmospheric and oceanic northward transport Tr vs. latitude Φ.

Solution

Find: Tr (W) vs. Φ (°).

Start with the spreadsheet results from the previous solved example. Then include eqs. (11.7 to 11.9). A sidewalk width of 5° latitude was used in the previous spreadsheet, which corresponds to Δy = 555 km.



Check: Units OK. Physics OK. Curve good.

Discussion: (1 PW = 1 petawatt = 10^{15} W).

Interpretation: At 30°N, the northward transport by fluid circulations is roughly 5 PW, while at 60°N it is about 2.5 PW. Hence, more energy is flowing into the midlatitudes than out, causing a net heating that compensates radiative cooling.

THERMAL WIND RELATIONSHIP

As sketched in Fig 11.7, there is a greater thickness between pressure surfaces in warm air than in cold (see eq. 1.18, the hypsometric equation). If a horizontal temperature gradient is present, it changes the tilt of pressure surfaces with increasing altitude. But geostrophic wind is proportional to the tilt of the pressure surfaces (eq. 9.20). Thus, geostrophic wind changes with altitude when temperature changes in the horizontal.

The relationship between horizontal temperature gradient and vertical gradient of geostrophic wind (U_g, V_g) is called the **thermal wind relationship**:

$$\frac{\Delta U_g}{\Delta z} \approx \frac{-g}{T_v \cdot f_c} \cdot \frac{\Delta T_v}{\Delta y} \qquad \bullet(11.10a)$$

$$\frac{\Delta V_g}{\Delta z} \approx \frac{g}{T_v \cdot f_c} \cdot \frac{\Delta T_v}{\Delta x} \qquad \bullet(11.10b)$$

where $g = 9.8\ \text{m·s}^{-2}$ is gravitational acceleration, T_v is the virtual temperature (in Kelvins, and nearly equal to the actual temperature if the air is fairly dry), and f_c is the Coriolis parameter. Note that the north-south temperature gradient alters the east-west geostrophic winds with height, and vice versa.

As it turns out, most of the atmosphere is nearly in geostrophic equilibrium. Thus, the change of actual wind speed with height is nearly equal to the change of the geostrophic wind.

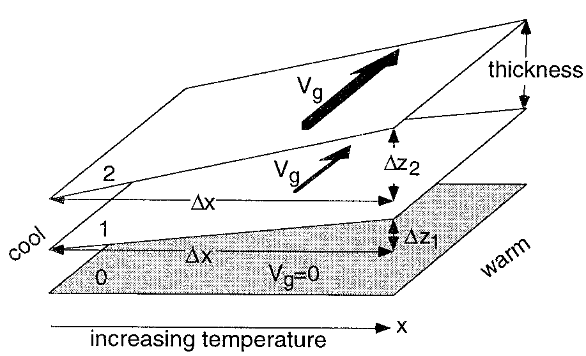


Figure 11.7
The three planes are surfaces of constant pressure (i.e., isobaric surfaces). Surface #2 has lower pressure than surface #1, etc. A horizontal temperature gradient tilts the pressure surfaces and causes the geostrophic wind to increase with height.

Solved Example

Suppose temperature increases from 8°C to 12°C toward the east, across a distance of 100 km (such as plotted in Fig 11.7). Find the vertical gradient of geostrophic wind, given $f_c = 10^{-4}\ \text{s}^{-1}$.

Solution

Assume: Dry air. Thus, $T_v = T$
Given: $\Delta T = 12 - 8°C = 4°C$, $\Delta x = 100$ km,
$T = 0.5·(8+12°C) = 10°C = 283$ K, $f_c = 10^{-4}\ \text{s}^{-1}$.
Find: $\Delta U_g/\Delta z$ & $\Delta V_g/\Delta z$ = ? (m/s)/km

Use eq. (11.10a):

$$\frac{\Delta U_g}{\Delta z} \approx \frac{-(9.8\text{m/s}^2)}{(283K)\cdot(10^{-4}\text{s}^{-1})}\cdot(0°\text{C/km}) = \underline{0}$$

Thus, U_g = uniform with height.
Use eq. (11.10b):

$$\frac{\Delta V_g}{\Delta z} \approx \frac{(9.8\text{m/s}^2)}{(283K)\cdot(10^{-4}\text{s}^{-1})}\cdot\frac{(4°\text{C})}{(100\text{km})}$$

$= \underline{\textbf{13.9 (m/s)/km}}$

Check: Units OK. Physics OK. Agrees with Fig.
Discussion: Each kilometer gain in altitude gives a 13.9 increase in northward geostrophic wind speed. For example, if the wind at the surface is –3.9 m/s (i.e., light from the north), then the wind at 1 km altitude is 10 m/s (strong from the south).

Thickness

The **thickness** between two different pressure surfaces is a measure of the average virtual temperature within that layer. For example, consider the two different pressure surfaces colored white in Fig 11.7. The air is warmer in the east than in the west. Thus, the thickness of that layer is greater in the east than in the west.

Maps of thickness are used in weather forecasting. One example is the "100 to 50 kPa thickness" chart, such as shown in Fig 11.8. This is a map with contours showing the thickness between the 100 kPa and 50 kPa isobaric surfaces. For such a map, regions of low thickness correspond to regions of cold temperature, and vise versa. It is a good indication of average temperature in the bottom half of the troposphere, and is useful for identifying air masses and fronts (discussed in the next chapter).

Define thickness TH as

$$TH = z_{P1} - z_{P2} \qquad (11.11)$$

where z_{P2} and z_{P1} are the heights of the P_2 and P_1 isobaric surfaces.

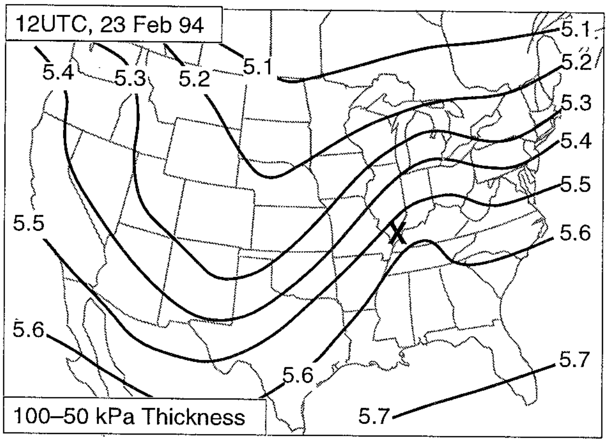


Figure 11.8
100–50 kPa thickness (km), valid at 12 UTC on 23 Feb 94. X marks sfc. low.

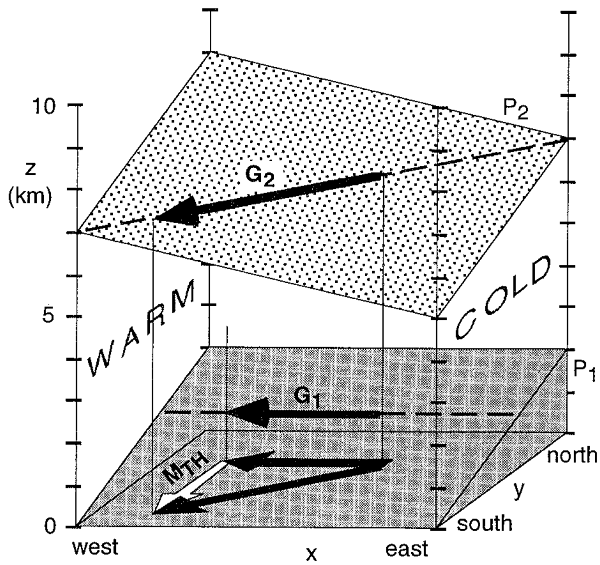


Figure 11.9
Given an isobaric surface P_1 (gray), with a height contour shown as the dashed line. The geostrophic wind G_1 on the surface is parallel to the height contour, with low heights to the left (N. Hem.). Isobaric surface P_2 (for $P_2 < P_1$) is also plotted (stippled). Cold air to the east is associated with a thickness of TH = 5 km between the two pressure surfaces. To the west, warm air has thickness 7 km. These thicknesses are added to the bottom pressure surface, to give the corners of the upper surface. On that upper surface are shown a height contour (dashed line) and the geostrophic wind vector G_2. We see that $G_2 > G_1$ because the top surface has greater slope than the bottom. The thermal wind is parallel to the thickness contours with cold to the left (white arrow). This thermal wind is the vector difference between the geostrophic winds at the two pressure surfaces, as shown in the projection on the ground.

Thermal Wind

The thermal wind relationship can be applied over the same layer of air bounded by isobaric surfaces as was used to define thickness. This gives:

$$U_{TH} \equiv U_{g2} - U_{g1} = -\frac{g}{f_c}\frac{\Delta TH}{\Delta y} \qquad \bullet(11.12a)$$

$$V_{TH} \equiv V_{g2} - V_{g1} = +\frac{g}{f_c}\frac{\Delta TH}{\Delta x} \qquad \bullet(11.12b)$$

where subscripts $g2$ and $g1$ denote the geostrophic winds on the P_2 and P_1 pressure surfaces, g is gravitational acceleration, and f_c is the Coriolis parameter.

The variables U_{TH} and V_{TH} are known as the **thermal wind** components. Taken together (U_{TH}, V_{TH}) they represent the vector difference between the geostrophic winds at the top and bottom pressure surfaces. Thermal wind magnitude M_{TH} is:

$$M_{TH} = \sqrt{U_{TH}^{\ 2} + V_{TH}^{\ 2}} \qquad (11.13)$$

In Fig 11.7, this vector difference happened to be in the same direction as the geostrophic wind. But in general, this is not usually the case, as illustrated in Fig 11.9.

Eqs. (11.12) imply that the thermal wind is parallel to the thickness contours, with cold temperatures (low thickness) to the left in the N. Hemisphere (Fig 11.10). Closer packing of the thickness lines gives stronger thermal winds, because the horizontal temperature gradient is larger there.

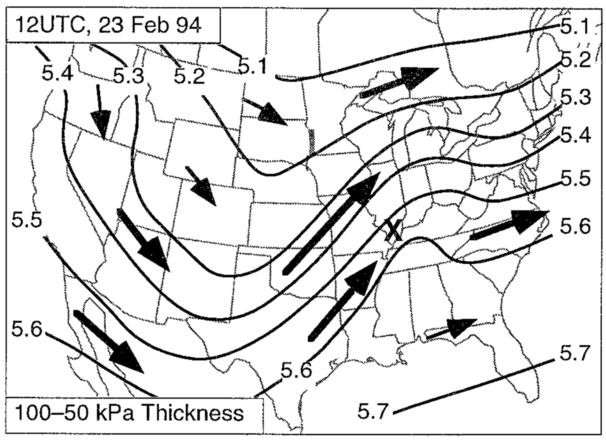


Figure 11.10
100–50 kPa thickness (km), with thermal wind vectors added. Larger vectors qualitatively denote stronger thermal winds.

Thermal winds on a thickness map behave analogously to geostrophic winds on a constant pressure or height map, making their behavior a bit easier to remember. However, while it is possible for actual winds to equal the geostrophic wind, there is no real wind that equals the thermal wind. The thermal wind is just the vector difference between geostrophic winds at two different heights or pressures.

Solved Example

Suppose the thickness of the 100 - 70 kPa layer is 2.9 km at one location, and 3.0 km at a site 500 km to the east. Find the components of the thermal wind vector, given $f_c = 10^{-4}$ s^{-1}.

Solution

Assume: No north-south thickness gradient.
Given: $TH_1 = 2.9$ km, $TH_2 = 3.0$ km,
$\Delta x =$ 500 km, $f_c = 10^{-4}$ s^{-1}.
Find: $U_{TH} = ?$ m/s, $V_{TH} = ?$ m/s

Use eq. (11.12a): $U_{TH} =$ **0 m/s**.
Use eq. (11.12b):

$$V_{TH} = \frac{g}{f_c} \frac{\Delta TH}{\Delta x} = \frac{(9.8\text{ms}^{-2}) \cdot (3.0 - 2.9)\text{km}}{(10^{-4}\text{s}^{-1}) \cdot (500\text{km})}$$
$$= \underline{19.6 \text{ m/s}}$$

Check: Units OK. Physics OK.
Discussion: There is no east-west thermal wind component because the thickness does not change in the north-south direction. The positive sign of V_{TH} means a wind from south to north, which agrees with the rule that the thermal wind is parallel to the thickness contours with cold air to the left (west, in this example).

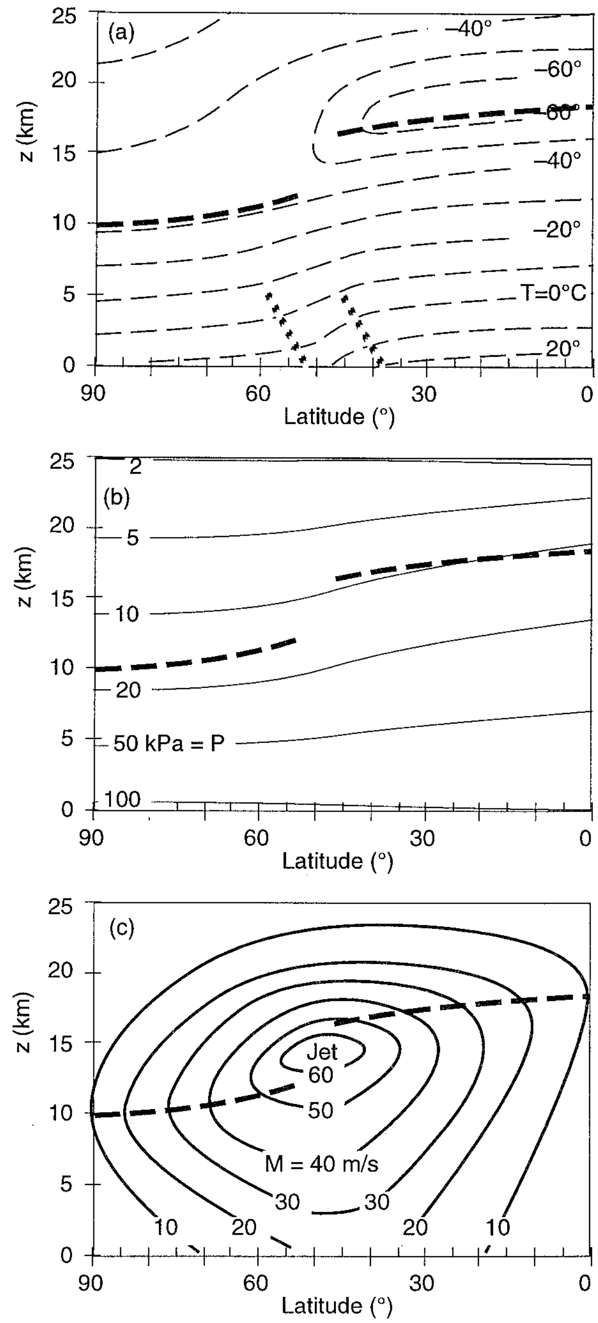


Figure 11.11
Vertical slices through the N. Hemisphere atmosphere: (a) isotherms, (b) isobars, (c) isotachs. Heavy dashed line marks the tropopause, and a frontal zone is between the dotted lines in (a). Wind direction in (c) is from the reader into the page. These figures can be overlain.

JET STREAM

Baroclinicity (i.e., the north-south temperature gradient) drives the west-to-east winds near the top of the troposphere, via the thermal wind relationship. Angular momentum issues will also be explored.

Baroclinicity

Fig 11.11a shows a typical isotherm pattern. Air near the ground is warmer near the equator, colder at the poles, and there is a frontal zone at mid-latitudes where temperature decreases rapidly toward the north. This north-south temperature gradient exists throughout the troposphere.

However, the tropopause is lower near the poles than near the equator. Thus, temperature begins increasing with height at a lower altitude near the poles than near the equator. This causes a temperature reversal in the stratosphere, where the air is colder over the equator and warmer over the poles. The spatial distribution of temperature is called the **temperature field**.

Apply the thermal wind equation to the temperature field to give the **pressure field** (the spatial distribution of pressures, Fig 11.11b). In the troposphere, greater thickness between pressure surfaces in the warmer equatorial air than the colder polar air causes the isobars to become more tilted at mid-latitudes as the tropopause is approached. Above the tropopause, tilt decreases because the north-south temperature gradient is reversed.

Regions with the greatest tilt have the greatest south-to-north pressure gradient, which drives the fastest geostrophic wind (Fig. 11.11c). This maximum of westerly winds, known as the **jet stream**, occurs at the tropopause in mid-latitudes. The center of the jet stream, known as the **core**, occurs in a region that can be idealized as a break or fold in the tropopause, as will be discussed in Chapter 12. Moderately strong winds also occur near the frontal zone.

The temperature model presented in eq. (11.1) and Fig 11.3 is a starting point for quantifying the nature of the jet stream. Equations (11.2), (11.3) and (11.10) can be combined to give the wind speed as a function of latitude ϕ and altitude z, where it is assumed for simplicity that the winds near the ground are zero:

$$U_{jet} \approx \frac{g \cdot c \cdot b_1}{2\Omega \cdot T_v} \cdot z \cdot \left(1 - \frac{z}{2 \cdot z_T}\right) \cdot \cos^2(\phi) \cdot \sin^2(\phi) \qquad (11.14)$$

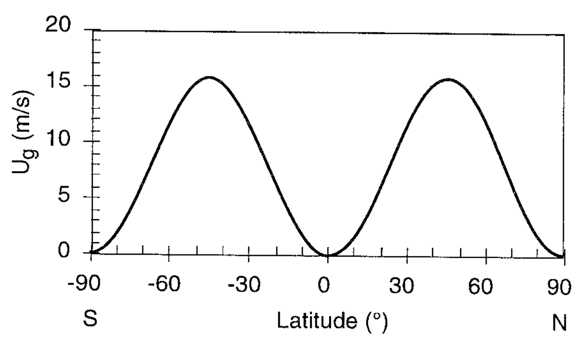


Figure 11.12
Geostrophic winds at 11 km altitude (idealized).

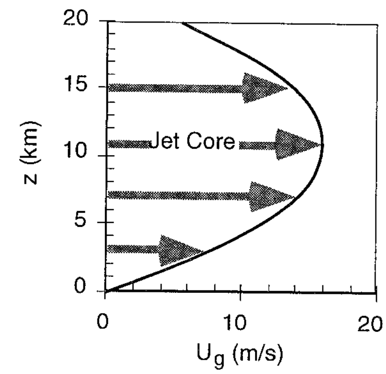


Figure 11.13
Vertical profile of (idealized) geostrophic wind at 45° latitude.

where U_{jet} is used in place of U_g to indicate a specific application of geostrophic wind to the polar jet stream, and where $c = 1.18\text{x}10^{-3}$ km^{-1} and $b_1 \cong 40$ K as before. The definition of the Coriolis parameter eq. (9.10) $f_c = 2 \cdot \Omega \cdot \sin\phi$ was used, where $2 \cdot \Omega =$ $1.458\text{x}10^{-4}$ s^{-1}. The average troposphere depth is $z_T \cong 11$ km, gravitation acceleration is $g = 9.8$ m/s^2, and T_v is the average virtual temperature (in Kelvins).

From Fig 11.3b we see there are two extrema of north-south temperature gradient, one in the northern hemisphere and one in the southern. Those are the latitudes where we can anticipate to find the strongest jet velocities (Fig 11.12). Although the temperature gradient in the southern hemisphere has a sign opposite to that in the north, the sign of the Coriolis parameter also changes. Thus, the jet stream velocity is positive (west to east) in both hemispheres.

A spreadsheet solution of eq. (11.11) is shown in Figs 11.12 to 11.14. At the tropopause (z = 11 km), Fig 11.12 shows the jet-stream velocity, with maxima at 45° north and south latitudes. For 45° latitude, Fig 11.13 shows the vertical profile of geostrophic wind speed, with the fastest speeds (i.e., the core of the jet) at the tropopause. Fig 11.14 shows a vertical cross-section (altitude vs. latitude).

The gross features shown in this figure are indeed observed in the atmosphere. However, actual spacing between jets in the northern and southern hemispheres is less than 90°; it is roughly 70° or 75° latitude difference. Also, the centers of both jets shift slightly northward during northern-hemisphere summer, and southward in winter. Jet speeds are not zero at the surface, and the jets tend to spread slightly toward the poles. Nevertheless, the empirical model presented here via eq. (11.1) serves as an instructive first introduction.

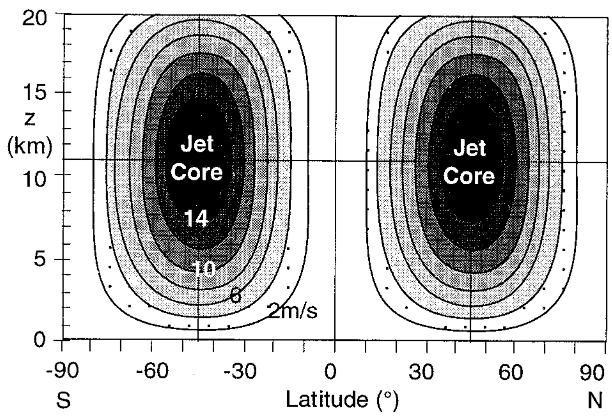


Figure 11.14
Idealized vertical cross-section of the atmosphere, showing isotachs.

Solved Example

Find the geostrophic wind speed at the tropopause at 45° latitude, assuming an idealized temperature structure as plotted in Fig 11.3. Assume an average temperature of 0°C, and neglect moisture.

Solution

Given: $z = z_T = 11$ km, $\phi = 45°$, $T_v = 273$ K.
Find: $U_g = ?$ m/s

Use eq. (11.14):

$$U_{jet} \approx \frac{(9.8\text{m}\cdot\text{s}^{-2})\cdot(1.18\times10^{-3}\text{km}^{-1})\cdot(40\text{K})}{(1.458\times10^{-4}\text{s}^{-1})\cdot(273\text{K})}\cdot (11\text{km})\cdot\left(1-\frac{1}{2}\right)\cdot\cos^2(45°)\cdot\sin^2(45°)$$

= <u>16 m/s</u>

Check: Units OK. Physics OK.
Discussion: Actual average wind speeds of about 40 m/s are observed in the jet stream of the winter hemisphere over a three-month average. Sometimes jet velocities as great as 100 m/s are observed on individual days. In the winter hemisphere, the temperature gradient is often greater than that given by eq. (11.3), which partially explains our low wind speed.

Angular Momentum

Another factor affecting the jet stream is conservation of angular momentum. This approach has serious deficiencies, which we will discuss.

Consider air initially at rest at some initial latitude such as the equator; namely, it is moving eastward with the earth's surface as the earth rotates about its axis. If there were no forces acting on the air, then it would preserve its eastward **angular momentum** as it moves northward from a source latitude ϕ_s to a destination latitude ϕ_d (Fig 11.15), then:

$$m\cdot U_s\cdot R_s = m\cdot U_d\cdot R_d \qquad \bullet(11.15)$$

where m is the mass of air, U_s is the tangential velocity of the earth at the source latitude, and U_d is the tangential velocity of the air from the source after it gets to the destination. The radius from the earth's axis to the surface at latitude ϕ is $R_\phi = R_e\cdot\cos(\phi)$, where R_ϕ represents source or destination radius (R_s or R_d), and R_e = 6357 km is the earth radius.

The tangential velocity of the earth at latitude ϕ is $U_\phi = \Omega\cdot R_\phi = \Omega\cdot R_e\cdot\cos(\phi)$, where the rate of rotation is $\Omega = 0.729\text{x}10^{-4}\ \text{s}^{-1}$. When an air parcel moves from a source to a destination, its velocity U' relative to the earth is

$$U' \approx \Omega\cdot R_e\cdot\left[\frac{\cos^2\phi_s}{\cos\phi_d} - \cos\phi_d\right] \qquad (11.16)$$

For example, Fig 11.16 shows the U' values of air reaching a destination of 45° latitude from sources at other latitudes.

Jet-stream winds of 100 - 300 m/s predicted by this method are not routinely observed on earth. In the real atmosphere, angular momentum is <u>not</u> conserved because pressure gradient, Coriolis, and turbulent drag forces act on the air.

For example, as a poleward moving parcel accelerates toward the east, increasing Coriolis force would turn it back toward the equator. Such turning causes wind to converge and alter the pressure

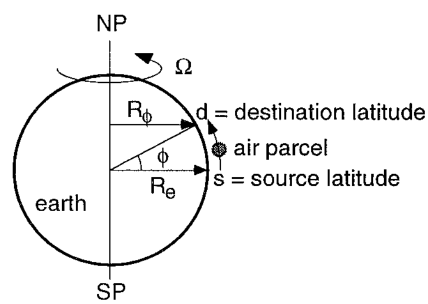


Figure 11.15
Northward air parcel movement.

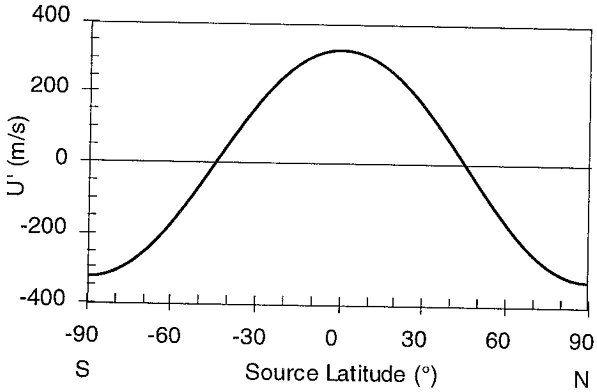


Figure 11.16
Relative wind speed at destination 45° latitude of air that came from various source latitudes.

gradient. The action of pressure-gradient, Coriolis, and turbulent drag forces prevents angular momentum from being conserved, and is one reason why the equator-to-pole circulation pattern of Fig 11.1 does not happen in the real atmosphere. The actual vertical circulation pattern extends only to about 30° north and south and is called the **Hadley cell**, as will be explained at the end of this chapter.

Poleward of 30° latitude, heat is still transferred poleward; however, this transfer is primarily by north-south meanders (**planetary waves**) of the jet stream rather than by a direct vertical circulation. Planetary waves are explained in the middle of this Chapter, using the concept of vorticity.

Solved Example

In the free atmosphere (no turbulence), what would be the relative velocity at 30°N for an air parcel that moved northward from the equator?

Solution

Given: $\phi_d = 30°N$, $\phi_s = 0°$. No turbulence.
Find: $U' = ?$ m/s

Use eq. (11.16):

$$U' \approx (7.29\times10^{-5}s^{-1})\cdot(6.357\times10^{6}m)\cdot\left[\frac{1}{\cos(30°)} - \cos(30°)\right] = \underline{134\text{ m/s}}$$

Check: Units OK. Physics OK.
Discussion: Winds in the real atmosphere are much less than this, where roughly 95% of the reduction is thought to be due to pressure gradient force, and 5% due to turbulent drag. These forces prevent conservation of angular momentum.

VORTICITY

Relative Vorticity

Relative vorticity ζ_r is a measure of the rotation of fluids about a vertical axis relative to the earth's surface. It is defined as positive in the counterclockwise direction. The unit of measurement of vorticity is inverse seconds.

The following two definitions are equivalent:

$$\zeta_r = \frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y} \qquad \bullet(11.17)$$

$$\zeta_r = -\frac{\Delta M}{\Delta n} + \frac{M}{R} \qquad \bullet(11.18)$$

where (*U*, *V*) are the (eastward, northward) components of the wind velocity, *R* is the radius of curvature traveled by a moving air parcel, *M* is the tangential speed along that circumference in a counterclockwise direction, and *n* is the direction pointing inward toward the center of curvature.

A physical interpretation for the first equation is sketched in Fig 11.17. If fluid travels along a straight channel, but has shear, then it also has vorticity because a tiny paddle wheel carried by the fluid would be seen to rotate. The second equation is interpreted in Fig 11.18, where fluid following a curved path also has vorticity, so long as radial shear of the tangential velocity does not cancel it.

A special case of the last equation is where the radial shear is just great enough so that winds at different radii sweep out identical angular velocities about the center of curvature. In other words, the fluid rotates as a solid body. For this case, the last equation reduces to

$$\zeta_r = \frac{2M}{R} \qquad \bullet(11.19)$$

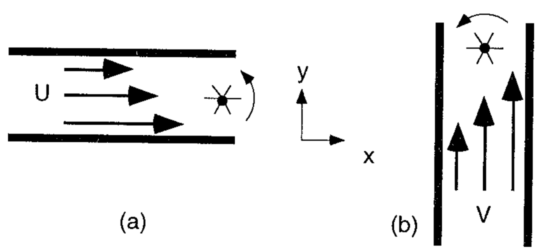


Figure 11.17
Shear-induced relative vorticity. Counter-clockwise turning of the paddle wheels indicates positive vorticity for both examples. (a) $\Delta U/\Delta y$ is negative. (b) $\Delta V/\Delta x$ is positive.

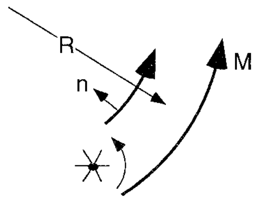


Figure 11.18
Vorticity for flow around a curve, where both the curvature and the shear contribute to positive vorticity in this example (i.e., the paddle wheel rotates counter-clockwise).

Solved Example
If wind rotates as a solid body about the center of a low pressure system, and the tangential velocity is 10 m/s at radius 300 km, find the relative vorticity.

Solution
Given: M = 10 m/s (cyclonic), R = 300,000 m.
Find: ζ_r = ? s^{-1}

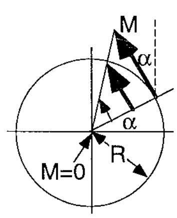
Use eq. (11.18):

$$\zeta_r = \frac{2\cdot(10\text{m/s})}{3\times10^5\text{m}} = \underline{6.67\text{x}10^{-5}}\ \text{s}^{-1}$$

Check: Units OK. Physics OK.
Discussion: Relative vorticities of synoptic storms are typically about this order of magnitude.

FOCUS • Solid-Body Relative Vorticity

One can derive eq. (11.19) from either eq. (11.17) or eq. (11.18). Consider the sketch at right, where speed M is tangent to the rotating disk at radius R.



Starting with eq. (11.18):

$$\zeta_r = -\frac{\Delta M}{\Delta n} + \frac{M}{R}$$

Thus:

$$\zeta_r = -\frac{(0-M)}{R} + \frac{M}{R} = \boxed{\frac{2M}{R}}$$

Or, starting with (11.17):

$$\zeta_r = \frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y} = \frac{(M\cos\alpha - 0)}{R\cos\alpha} - \frac{(-M\sin\alpha - 0)}{R\sin\alpha}$$

$$= \zeta_r = \frac{M}{R} + \frac{M}{R} = \boxed{\frac{2M}{R}}$$

Absolute Vorticity

Measured with respect to the "fixed" stars, the total vorticity must include the earth's rotation in addition to the relative vorticity. This sum is called the absolute vorticity ζ_a :

$$\zeta_a = \zeta_r + f_c \qquad \bullet(11.20)$$

where the Coriolis parameter $f_c = 2\Omega\cdot\sin(\phi)$ is a measure of the vorticity of the planet, and where 2Ω = 1.458x10^{-4} s^{-1} .

Potential Vorticity

Potential vorticity ζ_p is defined as the absolute vorticity divided by the depth Δz of the column of air that is rotating:

$$\zeta_p = \frac{\zeta_r + f_c}{\Delta z} = \text{constant} \qquad \bullet(11.21)$$

It has units of (m^{-1}·s^{-1}). In the absence of turbulent drag and heating (latent, radiative, etc.), **potential vorticity is conserved.**

Combining the previous equations yields:

$$\underbrace{\underbrace{-\frac{\Delta M}{\Delta n}}_{shear} + \underbrace{\frac{M}{R}}_{curvature}}_{relative} + \underbrace{f_c}_{planetary} = \underbrace{\zeta_p\cdot\Delta z}_{stretching} \qquad \bullet(11.22)$$

Solved Example
An 11 km deep layer of air at 45°N latitude has no curvature, but has a shear of –10 m/s across distance 500 km. What is the potential vorticity?

Solution
Assume the shear is in the cyclonic direction.
Given: Δz = 11000 m, ϕ = 45°N, ΔM = –10 m/s,
Δn = 500000 m.
Find: ζ_p = ? m^{-1}·s^{-1}

Use eq. (11.21):

$$\zeta_p = \frac{\frac{(10\text{m/s})}{5\times10^5\text{m}} + (1.458\times10^{-4}\text{s}^{-1})\cdot\sin(45°)}{11000\text{m}}$$

$= (2+10.31)\text{x}10^{-5}$ /11000 m $= \underline{1.12\text{x}10^{-8}}$ m^{-1}·s^{-1}

Check: Units OK. Physics OK.
Discussion: Planetary vorticity is 5 times greater than the relative vorticity for this case. It cannot be neglected when computing potential vorticity.

where ζ_p is a constant that depends on the initial conditions of the flow. The last term states that if the column of rotating air is stretched vertically, then its relative vorticity must increase or it must move further north where planetary vorticity is greater.

Isentropic Potential Vorticity

Isentropic potential vorticity (IPV) can be defined as

$$\zeta_{IPV} = \zeta_p \cdot \frac{\Delta\theta}{\rho} = \text{constant} \qquad (11.23a)$$

where ζ_p is the potential vorticity measured on an isentropic surface (i.e., a surface connecting points of equal potential temperature θ), and where ρ is air density.

By rewriting the previous equation as

$$\zeta_{IPV} = \frac{\zeta_r + f_c}{\rho} \cdot \left(\frac{\Delta\theta}{\Delta z}\right) \qquad (11.23b)$$

we see that larger IPVs exist where the air is less dense and where the static stability $\Delta\theta/\Delta z$ is greater. For this reason, the IPV is two orders of magnitude greater in the stratosphere than the troposphere.

IPV is measured in potential vorticity units (PVU), defined by $1\ \text{PVU} = 10^{-6}\ \text{K·m}^2\text{·s}^{-1}\text{·kg}^{-1}$. On the average, air in the troposphere has $\zeta_{IPV} < 1.5$ PVU, while in the stratosphere it is greater.

Isentropic potential vorticity is conserved for air moving adiabatically and frictionlessly along an isentropic surface (i.e., a surface of constant potential temperature). Thus, it can be used as a tracer of air. Stratospheric air entrained into the troposphere retains its IPV > 1.5 PVU for a while before losing its identity due to turbulent mixing.

Solved Example

Find the IPV for the previous example, using $\rho = 0.5\ \text{kg/m}^3$ and $\Delta\theta/\Delta z = 3.3$ K/km.

Solution

Given: $\zeta_p = 1.12\text{x}10^{-8}\ \text{m}^{-1}\text{·s}^{-1}$, $\rho = 0.5\ \text{kg/m}^3$, $\Delta\theta/\Delta z = 3.3$ K/km, $\Delta z = 11$ km

Find: $\zeta_{IPV} = ?$ PVU

Use eq. (11.23b):

$$\zeta_{IPV} = \frac{(1.12\times10^{-8}\text{m}^{-1}\text{s}^{-1})\cdot(11\text{km})}{(0.5\text{kg}\cdot\text{m}^{-3})} \cdot \left(3.3\text{K}\cdot\text{km}^{-1}\right)$$

$$= 8.13\text{x}10^{-7}\ \text{K·m}^2\text{·s}^{-1}\text{·kg}^{-1} = \underline{0.813\ \text{PVU}}$$

Check: Units OK. Physics OK.
Discussion: Reasonable value < 1.5 PVU in trop.

MIDLATITUDE TROUGHS AND RIDGES

The jet streams do not follow parallels in zonal flow around the earth to make perfect circles around the north and south poles. Instabilities in the atmosphere cause the jet stream to meander north and south in a wavy pattern as they encircle the globe (Fig. 11.19). These waves are called **planetary waves**, and have a wavelength of roughly 3000 to 4000 km. The number of waves around the globe is called the **zonal wavenumber**, and is typically 7 or 8, although they can range from about 3 to 13.

In Fig 11.19, jet stream winds (shaded) follow the height contours in a general counterclockwise (west to east) circulation as viewed looking down on the North Pole. The jet stream roughly demarks the boundary between the cold polar air from the warmer tropical air, because this temperature difference generates the jet stream winds due to the thermal wind relationship.

Regions of relatively low pressure or low height are called **troughs**. The center of the trough, called the **trough axis**, is indicated with a dashed line. Winds turn cyclonically (counterclockwise in the N. Hemisphere) around troughs; hence, troughs and low-pressure centers (low) are similar.

Ridges of relatively high pressures or heights are between the troughs. **Ridge axes** are indicated with a zig-zag line. Air turns anticyclonically (clockwise

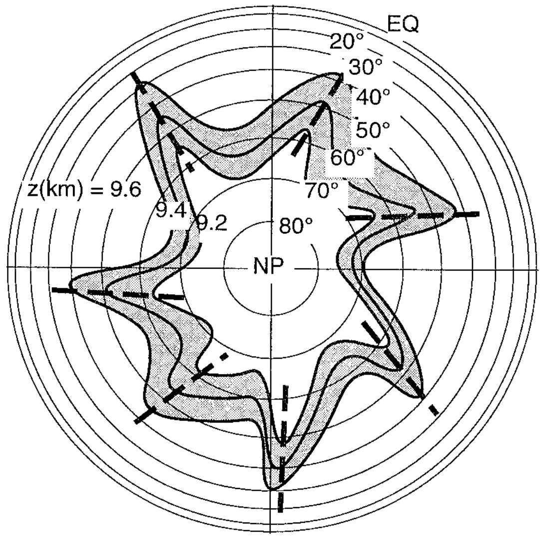


Figure 11.19
Height (z) contours of the 30 kPa pressure surface. View is looking down on the north pole (NP), and circles are latitude lines. Dashed lines indicate troughs. The jet stream is shaded.

in the N. Hemisphere) around ridges; hence, ridges and high-pressure centers (highs) are similar.

BEYOND ALGEBRA • Vorticity of a Wave

Problem: How does relative vorticity ζ_r vary with distance east (*x*) in a planetary wave?

Solution: Idealize the planetary wave as a sine wave

$$y = A \cdot \sin(2\pi \cdot x / \lambda) \qquad \text{(a)}$$

where *y* is distance north of some reference parallel (such as 45°N), *A* is north-south amplitude, and λ is east-west wavelength. Assume constant wind speed *M* following the path of the sine wave. The *U* and *V* components can be found from the slope *s* of the curve. Combining $s \equiv V / U$ & $U^2 + V^2 = M^2$ gives:

$$U = M \cdot (1 + s^2)^{-1/2} \text{ and } V = M \cdot s \cdot (1 + s^2)^{-1/2} \qquad \text{(b)}$$

But the slope is just the first derivative of eq. (a)

$$s = \partial y / \partial x = (2\pi A / \lambda) \cdot \cos(2\pi \cdot x / \lambda) \qquad \text{(c)}$$

Write eq. (11.17) in terms of partial derivatives:

$$\zeta_r = \partial V / \partial x - \partial U / \partial y \qquad \text{(d)}$$

which can be rewritten as:

$$\zeta_r = \partial V / \partial x - (\partial U / \partial x) \cdot (\partial x / \partial y) \quad \text{or}$$

$$\zeta_r = \partial V / \partial x - (\partial U / \partial x) \cdot (1 / s) \qquad \text{(e)}$$

Plugging eqs. (b) and (c) into (e) gives the answer:

$$\zeta_r = \frac{-2 \cdot M \cdot A \cdot \left(\frac{2\pi}{\lambda}\right)^2 \cdot \sin\left(\frac{2\pi x}{\lambda}\right)}{\left[1 + \left(\frac{2\pi A}{\lambda}\right)^2 \cdot \cos^2\left(\frac{2\pi x}{\lambda}\right)\right]^{3/2}} \qquad \text{(f)}$$

This is plotted in Fig a, for an example where λ = 4000 km, *A* = 1000 km, and *M* = 50 m/s.

Discussion: $|\zeta_r| = 2M / R$, where *R* is the radius of curvature for a sine wave, as is given in many calculus textbooks. Notice the narrow, sharp peaks of vorticity. Increased wave amplitude narrows the vort. peaks.

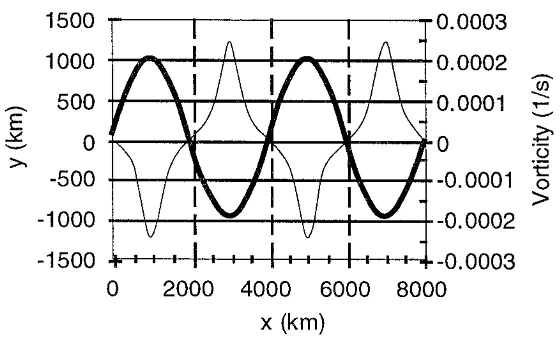


Figure a. *Streamline of an idealized planetary wave (thick), and the corresponding vorticity (thin).*

Two types of instabilities trigger these waves in the general circulation: barotropic instability, and baroclinic instability. Barotropic instability, caused by the earth's rotation, is described next. Baroclinic instability adds the effects of the north-south temperature gradient.

Barotropic Instability & Rossby Waves

Picture the jet stream at midlatitudes, blowing from west to east. If some small disturbance (such as flow over mountains, discussed in Chapter 13) causes the jet to turn slightly northward, then conservation of potential vorticity causes the jet to meander north and south. This meander of the jet stream is called a **Rossby wave** or **planetary wave**.

To understand this process, picture an initially-zonal flow at midlatitudes, such as sketched in Fig 11.20 at point 1. Straight zonal flow has no relative vorticity (no shear or curvature), but there is the planetary vorticity term in eq. (11.22) related to the latitude of the flow. For the special case of a fluid of fixed depth Δz such as the troposphere (Δz = 11 km), the conservation of potential vorticity simplifies to

$$\left[\frac{M}{R} + f_c\right]_{initial} = \left[\frac{M}{R} + f_c\right]_{later} \qquad (11.24)$$

where we will focus on the curvature term as a surrogate to the full relative vorticity.

If this flow is perturbed slightly (at point 2) to turn to the north, the air is now moving into higher latitudes where the Coriolis parameter and planetary vorticity are greater. Thus, a negative shear or curvature (negative *R*) must form in the flow to compensate the increased planetary shear, in order to keep potential vorticity constant. In plain words, the jet turns clockwise (**anticyclonic**) at point 3 until it points southeast.

As it proceeds southward toward its initial latitude (point 4), it has less curvature (i.e., less relative vorticity), but still points southeast. The jet then continues south of its initial latitude to a region where planetary vorticity is less (point 5). To preserve potential vorticity, it develops a **cyclonic** (counterclockwise) curvature and heads back northeast. Thus, initially stable (zonal) flow from point 1 has become wavy, and is said to have become **unstable**.

This Rossby wave requires a variation of Coriolis parameter with latitude to create the instability, which is called **barotropic instability**. Parameter $\beta \equiv \Delta f_c / \Delta y$ gives the rate of change of Coriolis parameter with latitude:

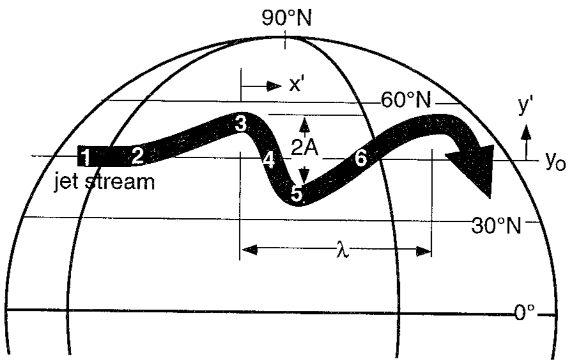


Figure 11.20
Initially zonal flow at point 1, if disturbed at point 2, will develop north-south meanders called Rossby waves.

$$\beta \equiv \frac{\Delta f_c}{\Delta y} = \frac{2 \cdot \Omega}{R_{earth}} \cdot \cos\phi \qquad \bullet(11.25)$$

where R_{earth} =6357 km is the average earth radius. Thus, $2 \cdot \Omega / R_{earth} = 2.29\text{x}10^{-11}\ \text{m}^{-1} \cdot \text{s}^{-1}$, and β is on the order of (1.5 to 2)x$10^{-11}\ \text{m}^{-1} \cdot \text{s}^{-1}$.

The path taken by the wave is approximately:

$$y' \approx A \cdot \cos\left[2\pi \cdot \left(\frac{x' - c \cdot t}{\lambda}\right)\right] \qquad (11.26)$$

where y' is the north-south displacement distance from the center latitude Y_o of the wave, x' is the distance east from some arbitrary longitude, c is the phase speed (the speed at which the crest of the wave moves relative to the earth), A is the north-south amplitude of the wave, and λ is the wavelength (see Fig 11.20).

Typical wavelengths are $\lambda \cong 6000$ km, although a wide range of wavelengths can occur. The circumference ($2\pi R_{earth} \cdot \cos\phi$) along a parallel at midlatitudes limits the total number of barotropic waves that can fit around the globe to about 4 to 5. The north-south domain of the wave roughly corresponds to the 30° width of midlatitudes where the jet stream is strongest, giving $A \cong 1665$ km.

These waves propagate relative to the mean zonal wind U_o at **intrinsic phase speed** c_o of about

$$c_o = -\beta \cdot \left(\frac{\lambda}{2\pi}\right)^2 \qquad \bullet(11.27)$$

where the negative sign indicates westward propagation relative to the mean background flow. However, when typical values for β and λ are used in eq. (11.27), the intrinsic phase speed is found to be roughly half the magnitude of the eastward jet-stream speed.

A **phase speed** c relative to the ground is defined as:

$$c = U_o + c_o \qquad \bullet(11.28)$$

which gives the west-to-east movement of the wave crest. For typical values of c_o and background zonal wind speed U_o, the phase speed is positive. Hence, the mean wind pushes the waves toward the east relative to observers on the earth. Eastward moving waves are indeed observed.

BEYOND ALGEBRA • The Beta Plane

One can derive β from the definition for f_c. Starting with eq. (9.10): $f_c = 2\ \Omega \sin\phi$, take the derivative with respect to distance north:

$$\beta \equiv \frac{\partial f_c}{\partial y} = \frac{\partial f_c}{\partial \phi} \cdot \frac{\partial \phi}{\partial y} = \frac{\partial(2\Omega \sin\phi)}{\partial \phi} \cdot \frac{\partial \phi}{\partial y}$$

$$\beta = 2\Omega \cos\phi \cdot \frac{\partial \phi}{\partial y} \qquad (a)$$

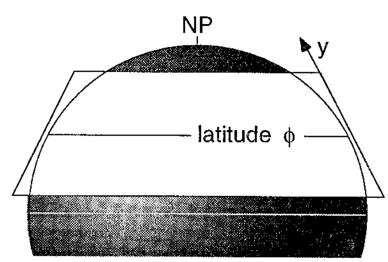
As was discussed in an earlier "Beyond Algebra" box, $\partial\phi / \partial y$ is the change of latitude per distance traveled north. The total change in latitude to circumnavigate the earth pole to pole is 2π radians, and the circumference of the earth is $2\pi R$ where R = 6356.766 km is the radius of the earth. Thus:

$$\frac{\partial \phi}{\partial y} = \frac{2\pi}{2\pi \cdot R} = \frac{1}{R} \qquad (b)$$

Plugging eq. (b) into (a) gives the desired answer:

$$\boxed{\beta = \frac{2 \cdot \Omega}{R_{earth}} \cdot \cos\phi} \qquad (11.25)$$

For midlatitude planetary waves confined to a latitude belt such as sketched in Fig 11.19, it is often convenient to assume β = constant. This has the same effect as assuming that a portion of the earth is shaped like a cone (as sketched below in white) rather than a sphere. The surface of the cone is called a **beta plane**, and looks like a lamp shade. Such an idealization allows barotropic effects to be described with slightly simpler math.



Equation (11.27) is called a **dispersion relation**, because waves of different wavelengths propagate at different phase speeds. Namely, waves that initially coincide would tend to separate or disperse with time.

By combining eqs. (11.27) and (11.28), waves of shorter wavelength (**short waves**) travel faster toward the east than **long waves**. Thus, short waves ride the long wave analogous to a car driving along a hilly road.

Solved Example(§)

A background jet stream of speed 50 m/s meanders with 6000 km wavelength and 1500 km amplitude, centered at 45°N. Plot the path (i.e., the meridional displacement) of the **barotropic** wave between 0 ≤ x' ≤ 10000 km at some initial time and 6 hours later, and find the phase speed.

Solution

Given: $\phi = 45°$, $U_o = 50$ m/s, $\lambda = 6000$ km, $A = 1500$ km, $t = 0$ & 6 h

Find: $\beta = ?$ m^{-1}·s^{-1}, $c = ?$ m/s, $y'(x) = ?$ km

Use eq. (11.25):

$$\beta = \frac{1.458\times10^{-4}\,\text{s}^{-1}}{6378000\text{m}}\cdot\cos(45°) = \underline{\mathbf{1.62x10^{-11}}}\ \text{m}^{-1}\cdot\text{s}^{-1},$$

Use eq. (11.27):

$$c_o = -(1.62\times10^{-11}\text{m}^{-1}\text{s}^{-1})\left(\frac{6\times10^6\text{m}}{2\pi}\right)^2 = \underline{-14.7}\ \text{m/s}$$

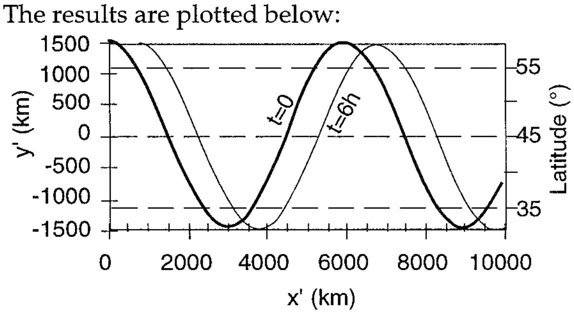
Use eq. (11.28):

$c = 50 - 14.7$ m/s = $\underline{\mathbf{35.26\ m/s}}$

Use eq. (11.26) on a spreadsheet for $t = 0$ & 6 h:

$$y' \approx (1500\text{km})\cdot\cos\left[2\pi\cdot\left(\frac{x' - (35.26\text{m/s})\cdot t}{6\times10^6\text{m}}\right)\right]$$

The results are plotted below:



Check: Units OK. Physics OK.

Discussion: The barotropic long wave propagates east about 800 km during the 6 h interval. The jet stream blows along this wavy path at speed 50 m/s.

Solved Example(§)

Same as the previous solved example, except that in addition to the previous long wave, there is also a **barotropic** short wave of 1000 km wavelength with amplitude 300 km.

Solution

Given: Same, plus $\lambda = 1000$ km, $A = 300$ km.

Find: $c = ?$ m/s, $y'(x') = ?$ km

Use eq. (11.27):

$$c_o = -(1.62\times10^{-11}\text{m}^{-1}\text{s}^{-1})\left(\frac{10^6\text{m}}{2\pi}\right)^2 = \underline{\mathbf{-0.41\ m/s}}$$

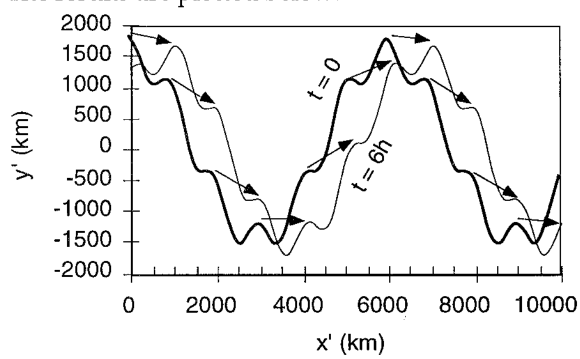
Use eq. (11.28)

$c = 50 - 0.41$ m/s = $\underline{\mathbf{49.6\ m/s}}$

Use eq. (11.26):

$$y' \approx (300\text{km})\cdot\cos\left[2\pi\cdot\left(\frac{x' - (49.6\text{m/s})\cdot t}{1\times10^6\text{m}}\right)\right]$$

The results are plotted below:



Check: Units OK. Physics OK.

Discussion: The movement of each short wave is indicated with the arrows. Not only do they move with the propagating background long wave, but they also ride the wave toward the east at a speed nearly equal to the wind speed.

Baroclinic Instability & Planetary Waves

Recall from Fig 11.1b that at midlatitudes the cold polar air slides under the warmer tropical air. This causes the air to be statically stable, as can be quantified by a Brunt-Väisälä frequency N_{BV}. The development later in this section extends what we learned of barotropic flows to the more complete **baroclinic** case having both β and N_{BV} effects in an environment with north-south temperature gradient.

First, we take a heuristic approach, and idealize the atmosphere as being a two layer fluid, with a north-south sloping density interface such as idealized in Fig 11.21. This will give us a qualitative picture of baroclinic waves.

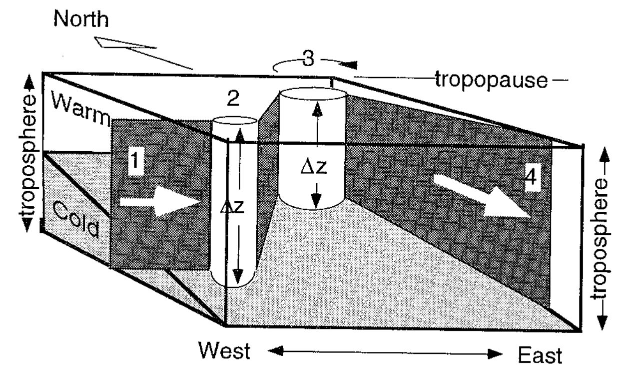


Figure 11.21
Some processes involved in baroclinic instability. Dark gray ribbon represents the jet-stream axis, while white columns indicate the absolute vorticity of the jet.

Qualitative Model

Initially zonal flow of the jet stream (sketched as point 1) has no relative vorticity, but it does have planetary vorticity related to its latitude. The narrow white column in the front left of Fig 11.21 represents air having such absolute vorticity.

If this jet stream is perturbed northward by some outside influence such as a mountain, it rides up on the density surface. The stratosphere is so statically stable that it acts like a lid on the troposphere. Thus, as air meanders northward, it is squeezed between the tropopause and the rising density interface. Namely, Δz shrinks. For this situation, the potential vorticity equation can be written as:

$$\left[\frac{f_c + (M/R)}{\Delta z}\right]_{initial} = \left[\frac{f_c + (M/R)}{\Delta z}\right]_{later} \qquad (11.29)$$

The column depth is less at point 3 than initially at point 2, hence the absolute vorticity at 3 must also be less than at 2, in order for the ratio of absolute vorticity to depth to remain constant. The planetary contribution doesn't help – in fact it is larger at point 3 as discussed in the previous subsection. Hence, the only way to conserve potential vorticity is for the relative vorticity to decrease substantially. As it decreases below its initial value of zero, the jet-stream path curves anticyclonically at point 3.

The jet stream overshoots to the south, and develops cyclonic relative vorticity and turns back to the north. The resulting breakdown of zonal flow into wavy flow is called **baroclinic instability**. The waves look similar to those in Fig 11.20, except with shorter wavelength because now both β and Δz work together to cause the oscillation.

Quantitative Approach

To extend this argument, consider the continuous stratification of Fig 6.11. Between the 290 and 340 K isentropes, for example, the air is statically stable. A column of air bounded at the top bottom and top by these isentropes near the equator shortens as it moves poleward. Thus, we could use the Brunt-Väisälä frequency and the north-south temperature gradient (i.e., the baroclinicity) as a better representation of the physics than the two layer model of Fig 11.21.

Skipping a long derivation, we end up with a north-south displacement of a baroclinic wave that is approximately:

$$y' \approx A \cdot \cos\left(\pi \cdot \frac{z}{Z_T}\right) \cdot \cos\left[2\pi \cdot \left(\frac{x' - c \cdot t}{\lambda}\right)\right] \qquad (11.30)$$

where $Z_T \cong 11$ km is the depth of the troposphere, and A is north-south amplitude. The extra cosine term containing height z means that the north-south wave amplitude first decreases with height from the surface to the middle of the troposphere, and then increases with opposite sign toward the top of the troposphere. Namely, the planetary wave near the tropopause is 180° out of phase compared to that near the ground.

The dispersion relation is:

$$c_o = \frac{-\beta}{\pi^2 \cdot \left[\frac{4}{\lambda^2} + \frac{1}{\lambda_R^2}\right]} \qquad \bullet(11.31)$$

and

$$c = U_o + c_o \qquad \bullet(11.32)$$

Again, intrinsic phase speed c_o is negative with magnitude less than the background jet velocity U_o, allowing the waves to move east relative to the ground.

An "internal" **Rossby radius of deformation** λ_R is defined as:

$$\lambda_R = \frac{N_{BV} \cdot Z_T}{f_c} \qquad \bullet(11.33)$$

where f_c is the Coriolis parameter, and N_{BV} is the Brunt-Väisälä frequency. This radius relates buoyant and inertial forcings. It is on the order of 1300 km.

As for barotropic waves, baroclinic waves have a range of wavelengths that can exist in superposition. However, some wavelengths grow faster than others, and tend to dominate the flow field. For baroclinic waves, this dominant wavelength is on the order of 3000 to 4000 km, and is given by:

$$\lambda \approx 2.38 \cdot \lambda_R \qquad (11.34)$$

FOCUS • Rossby Deformation Radius and Geostrophic Adjustment - Part 1

Definitions:

The spatial distribution of wind speeds and directions is known as the **wind field.** Similarly, the spatial distribution of temperature is called the **temperature field.**

The **mass field** is the spatial distribution of air mass. As discussed in Chapter 1, pressure is a measure of the mass of air above. Thus, the term "mass field" generically means the spatial distribution of pressure (i.e., the **pressure field**) on a constant altitude chart, or of heights (the **height field**) on an isobaric surface.

In baroclinic conditions, the hypsometric equation allows changes to the pressure field to be described by changes in the temperature field. Similarly, the thermal wind relationship relates changes in the wind field to changes in the temperature field.

Geostrophic Adjustment:

In regions of the atmosphere where pressure gradient and Coriolis forces dominate, the atmosphere tends to adjust itself to be in geostrophic balance. This is called **geostrophic adjustment**, and is described quantitatively in the next chapter.

When a disturbance or change is imposed in either the wind field or the pressure field, the other field adjusts to reach a new geostrophic balance. These adjustments are strongest near the disturbance, and gradually weaken with distance. The e-folding distance, beyond which the disturbance is felt only a little, is called the **Rossby radius of deformation,** λ_R.

For example, if winds are forced to increase above their initial geostrophic value at one location, then these winds will redistribute air mass in the horizontal, modifying the pressure field until it reaches a new geostrophic balance with the imposed winds. Thus, **the mass field adjusts to the wind field.**

Alternately, if air mass is added over a particular location, then the modified pressure gradient will force air outward from the disturbance. On the way out, Coriolis force will turn the winds to the right (N. Hem.), until a new geostrophic wind field is achieved. Thus, **the wind field adjusts to the mass field.**

In the real atmosphere, both fields adjust toward a new geostrophic balance. For large scale disturbances ($\lambda > \lambda_R$), most of the adjustment is in the wind field. For small scale disturbances ($\lambda < \lambda_R$), most of the adjustment is in the temperature or pressure fields.

Solved Example(§)

Same as the previous solved example, except in a **baroclinic** case in a standard atmosphere for the one dominant wave. Wave amplitude is 1500 km. Plot results at t = 0 and 6 h for z = 0 and 11 km.

Solution

Given: Same, and use Table 1-4 for std. atmos:

$\Delta T = -56.5-15 = -71.5°C$ across Δz = 11 km.

$T_{avg} = 0.5\cdot(-56.5+15°C) = -20.8°C = 252$ K

Find: λ_R=? km, λ=? km, c=? m/s, $y'(x')$=? km

Use eq. (6.5a):

$$N_{BV} = \sqrt{\frac{(9.8\text{m/s})}{255\text{K}}\left(\frac{-65°\text{K}}{10^4\text{m}} + 0.0098\frac{\text{K}}{\text{m}}\right)} = \underline{0.0113}\ \text{s}^{-1}$$

Use eq. (9.10):

$$f_c = (1.458\times10^{-4}s^{-1})\cdot\sin(45°) = \underline{1.031\text{x}10^{-4}}\ \text{s}^{-1}$$

Use eq.(11.33):

$$\lambda_R = \frac{(0.0113\text{s}^{-1})\cdot(11\text{km})}{1.03\times10^{-4}\text{s}^{-1}} = \underline{1206\text{ km}}$$

Use eq. (11.31):

$$\lambda = 2.38\cdot\lambda_R = \underline{2870\text{ km}}$$

Use eq. (11.31):

$$c_o = \frac{-(1.62\times10^{-11}\text{m}^{-1}\text{s}^{-1})}{\pi^2\cdot\left[\dfrac{4}{(2870\text{km})^2} + \dfrac{1}{(1206\text{km})^2}\right]} = \underline{-1.40\text{ m/s}}$$

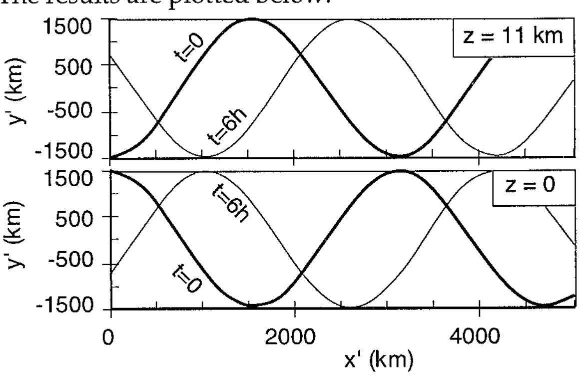
Use eq. (11.32)

$$c = 50 - 1.40\text{ m/s} = \underline{48.6\text{ m/s}}$$

Use eq. (11.30):

$$y' \approx (1500\text{km})\cos\left(\frac{\pi\cdot z}{11\text{km}}\right)\cos\left[2\pi\left(\frac{x' - 48.6\text{m/s}\cdot t}{2870\text{km}}\right)\right]$$

The results are plotted below:



Check: Units OK. Physics OK.

North-south displacement y' is not the only characteristic that is wavy in baroclinic flow. Also wavy are the perturbation velocities (u', v', w'), pressure p', potential temperature θ', and vertical displacement η', where the prime denotes deviation from the mean background state.

To present the wave equations for these other variables, we will simplify the notation by using:

$$a \equiv \pi\cdot z / Z_T \qquad (11.35)$$

and

$$b \equiv 2\pi\cdot(x - c\cdot t)/\lambda \qquad (11.36)$$

Thus:

$$
\begin{aligned}
y' &= \hat{Y}\cdot\cos(a)\cdot\cos(b) \\
\eta' &= \hat{\eta}\cdot\sin(a)\cdot\cos(b) \\
\theta' &= -\hat{\theta}\cdot\sin(a)\cdot\cos(b) \\
p' &= \hat{P}\cdot\cos(a)\cdot\cos(b) \\
u' &= \hat{U}\cdot\cos(a)\cdot\cos(b) \\
v' &= -\hat{V}\cdot\cos(a)\cdot\sin(b) \\
w' &= -\hat{W}\cdot\sin(a)\cdot\sin(b)
\end{aligned}
\qquad (11.37)
$$

Symbols wearing the caret hat (^) represent amplitude of the wave. These amplitudes are defined to be always positive (remember c_o is negative) in the Northern Hemisphere:

$$
\begin{aligned}
\hat{Y} &= A \\
\hat{\eta} &= \frac{A\cdot\pi\cdot f_c\cdot(-c_o)}{Z_T}\cdot\frac{1}{N_{BV}^2} \\
\hat{\theta} &= \frac{A\cdot\pi\cdot f_c\cdot(-c_o)}{Z_T}\cdot\frac{\theta_o}{g} \\
\hat{P} &= A\cdot\rho_o\cdot f_c\cdot(-c_o) \\
\hat{U} &= \left[\frac{A\cdot 2\pi\cdot(-c_o)}{\lambda}\right]^2\cdot\frac{1}{A\cdot f_c} \\
\hat{V} &= \frac{A\cdot 2\pi\cdot(-c_o)}{\lambda} \\
\hat{W} &= \frac{A\cdot 2\pi\cdot(-c_o)}{\lambda}\cdot\frac{\pi\cdot(-c_o)\cdot f_c}{Z_T\cdot N_{BV}^2}
\end{aligned}
\qquad (11.38)
$$

Each of these amplitudes depends on A, the amplitude of the meridional displacement, which is not fixed but depends on the initial disturbance.

Fig 11.22 illustrates the structure of this baroclinic wave in the Northern Hemisphere. Fig 11.22c corresponds to the surface weather maps used in the previous solved examples. All the variables are interacting to produce the wave. The impact of all these variables on synoptic weather will be discussed in Chapter 13.

Actual variables are found by adding the perturbation variables (eq. 11.37) to the background state. Mean background pressure P_o decreases hydrostatically with height, making the actual pressure at any height be $P = P_o + p'$.

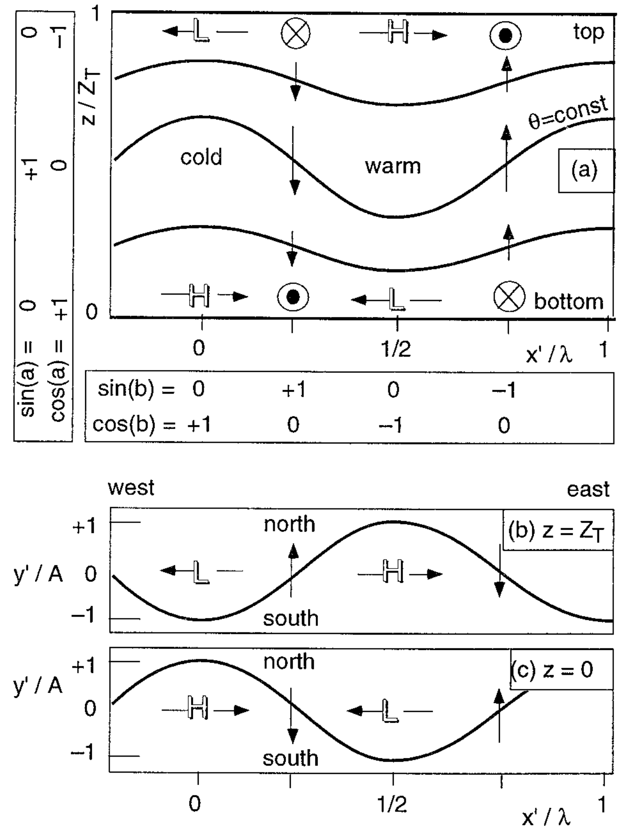


Figure 11.22
Idealized structure of baroclinic wave. (a) Vertical cross section. (b) Weather map at tropopause. (c) Surface weather map. Legend: Arrows indicate wind direction and speed, L and H indicate low and high pressure centers, dot-circle is southward-pointing vector, x-circle is northward pointing vector. Wavy lines on the cross section are isentropes. Also shown are sine and cosine terms from baroclinic wave equations. (after Cushman-Roisin, 1994)

Mean background potential temperature θ_o increases linearly with height (assuming constant N_{BV}), making the actual potential temperature $\theta = \theta_o + \theta'$. Background meridional and vertical winds are assumed to be zero, leaving $V = v'$, and $W = w'$. Background zonal wind U_o is assumed to be geostrophic and constant, leaving $U = U_o + u'$.

Many other factors affect wave formation in the jet stream, including turbulent drag, clouds and latent heating, and nonlinear processes in large-amplitude waves. Also, the more complete solution to baroclinic instability includes waves propagating meridionally as well as zonally (we focused on only zonal propagation here). However, our description above captured most of the important aspects of midlatitude flow that are observed in the real atmosphere.

Solved Example
Same as previous solved example, but find the pressure amplitudes at sea level.

Solution
Given: A = 1500 km, f_c = 9.47x10^{-5} s^{-1},
c_o = –1.66 m/s, ρ_o = 1.225 kg·m^{-3} .
Find: $\hat{P}$ = ? kPa

Use eq. (11.38):

$$\hat{P} = (1500\text{km})\left(1.225\frac{\text{kg}}{\text{m}^3}\right)(9.47\times10^{-5}\text{s}^{-1})\left(1.66\frac{\text{m}}{\text{s}}\right)$$

= **0.29 kPa**

Check: Units OK. Physics OK.
Discussion: Between high and low pressure is a distance of half a wavelength, which is 1564 km for this case. The pressure difference between crest and trough of the wave is 2·$\hat{P}$ = 0.58 kPa. Thus, the pressure gradient is 0.58/1564 = 0.00037 kPa/km, which is sufficient to drive the winds.
In the real atmosphere, condensation in clouds (which was neglected in eqs. 11.37) contributes to the dynamics to make horizontal pressure gradients even stronger.

Heat Transport

As the jet stream blows along the meandering planetary-wave path (Fig 11.23), it picks up warm air (T' = deviation from mean temperature = positive) and carries it northward (v' = +). Similarly, cold air (T' = negative) is carried southward (v' = –). The average meridional kinematic heat flux is the sum of advective transports divided by the number N of these transports: $F_{y\,waves}$ = (1/N)·Σ(v'·T') = (+)·(+) + (–)·(–) = positive. Thus, the north-south meandering jet stream causes a northward heat flux, $F_{y\,waves}$, without requiring a vertical circulation cell.

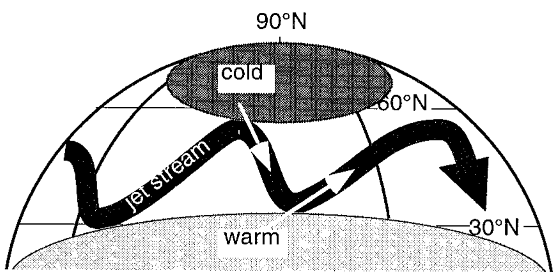


Figure 11.23
Heat transport by planetary waves in midlatitudes.

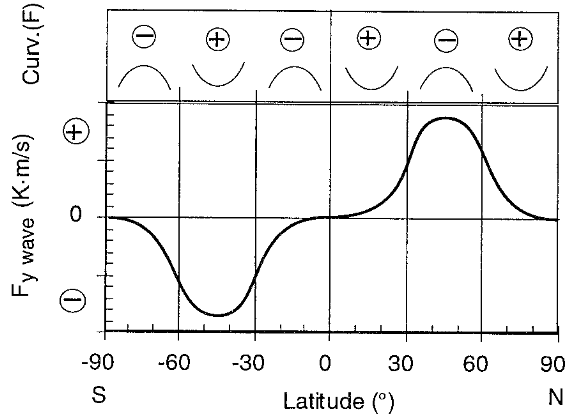


Figure 11.24
Bottom: Sketch of meridional wave flux of heat (positive northward, in kinematic units). Top: Sketch of sign of curvature of the wave flux.

The greatest flux is likely to occur where the waves have the most intense v-component (Fig. 11.23), and also where the north-south temperature gradient is greatest (Fig 11.4). Both of these processes conspire together to create large wave heat fluxes centered in midlatitudes (Fig 11.24 bottom).

Curvature (*Curv*) of a line is defined to be positive when the the line bends concave up (shaped like a bowl). Likewise, curvature is negative for concave down (shaped like a hill). The sign of the curvatures of wave heat flux are indicated in the top of Fig 11.24, which we will use in the next section.

Momentum Transport

Recall from Fig 11.16 that tropical air is faster (u' = positive), and polar air is slower (u' = negative) when moved to 45° latitude, because of angular-momentum conservation. The polar and tropical regions serve as reservoirs of various momentum that can be tapped by the meandering jet stream.

Analogous to heat transport, we find that the north-moving (v' = +) portions of the wave carry fast zonal momentum (u'= +) air northward, and south-moving (v' = –) portions carry slow zonal momentum (u' = –) southward (Fig 11.25). The net meridional transport of zonal momentum $\overline{u'v'}$ is positive, and is largest at the center of midlatitudes.
$\overline{u'v'}$ = (1/N)·Σ(v'·u') = (+)·(+) + (–)·(–) = positive.

However, the reservoir of tropical momentum is much larger than polar momentum because of the larger circumference of latitude lines there. Hence, tropical momentum can have more influence on mid-latitude air. To account for such unequal influence, a weighting factor $a = \cos(\phi_s)/\cos(\phi_d)$ can be included in the angular momentum expression:

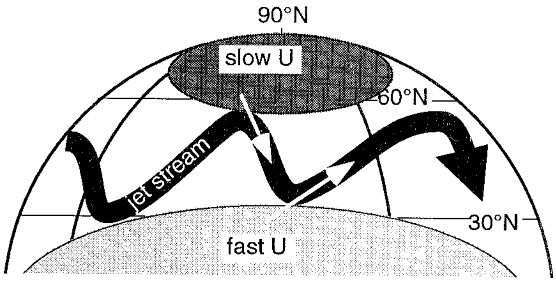


Figure 11.25
Meridional transport of eastward momentum by planetary waves in midlatitudes.

$$a \cdot u' \approx \Omega \cdot R_e \cdot \left[\frac{\cos^2 \phi_s}{\cos \phi_d} - \cos \phi_d \right] \cdot \frac{\cos \phi_s}{\cos \phi_d} \qquad (11.39)$$

Fig 11.26 shows that the weighted zonal velocity is asymmetric about 45° latitude. Namely, from 60° latitude the velocity is –100 m/s, but from a source of 30° latitude the velocity is about +200 m/s. Thus, although the momentum transport is positive everywhere, it decreases to the north. Hence, the meridional gradient MG of zonal momentum is negative in the N. Hemisphere mid-latitudes:

$$MG = \Delta \overline{u'v'} \ / \ \Delta y = \text{negative.} \qquad (11.40)$$

Although quantitatively the wind magnitudes from angular-momentum arguments are too large as discussed earlier, the qualitative picture of momentum transport is valid.

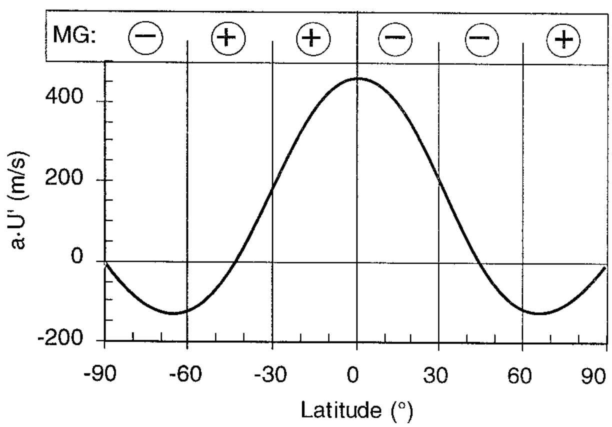


Figure 11.26
Zonal velocity from various source latitudes reaching destination 45°, weighted by the relative amounts of air in the source regions.

THREE-BAND GENERAL CIRCULATION

The preceding sections laid the groundwork to explain why there are three latitude-bands of circulation (Fig 11.27) between the equator and a pole, rather than one big Hadley cell. Namely, on earth we observe **direct** vertical circulation cells in the tropics (0° to 30°) and polar regions (60° to 90°), and an **indirect** cell at midlatitudes (30° to 60°). Direct means circulating in the same sense as the **Hadley cell**, with air rising at low latitudes and sinking at higher latitudes (Fig 11.28).

A measure of vertical cell circulation is:

$$CC = \frac{f_c^2}{N_{BV}^2} \frac{\Delta V}{\Delta z} - \frac{\Delta w}{\Delta y} \qquad (11.41)$$

CC is positive for direct cells in the N. Hemisphere, as illustrated in Fig 11.28. In a direct cell, vertical velocity decreases and even changes sign toward the north, making $\Delta w / \Delta y$ negative. Also, northward velocity increases with height, making $\Delta V / \Delta z$ positive for a direct cell. Both of these terms contribute to a positive CC in a direct cell. Similarly, CC is negative for an indirect cell.

The equations of motion can be combined to yield an equation for the cell circulation CC, which is written in abbreviated form as eq. 11.42. Buoyancy associated with a heated surface can drive a vertical circulation, as can momentum transport. But if some of the heating difference is transported by planetary waves, then there will less heat available to drive a direct circulation.

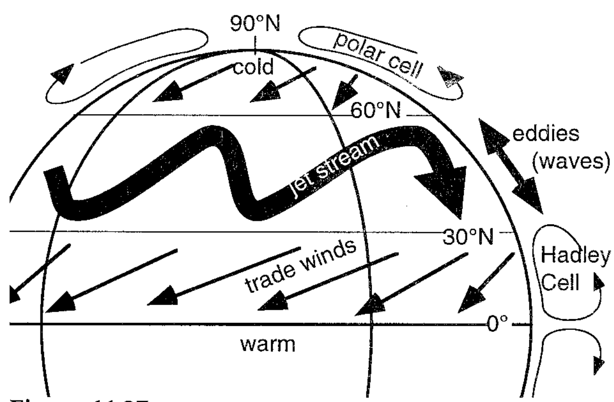


Figure 11.27
Three-band structure of general circulation: 1) vertical Hadley cell in tropics, 2) horizontal planetary waves at midlatitudes, and 3) a weak vertical circulation near the poles.

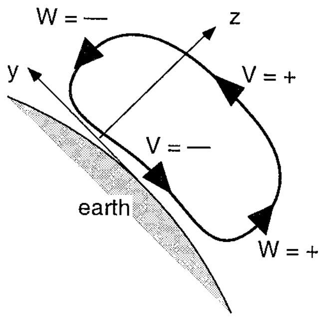


Figure 11.28
Definition of a direct circulation cell in the N. Hemisphere.

Based on the previous discussions of radiative differential heating E_{net}, momentum gradient MG, and heat flux curvature $Curv(F_{y\ wave})$, the contributions of each term to the circulation are listed below the equation. The sign of $\Delta MG/\Delta z$ equals the sign of MG, assuming that northward momentum transport is weakest at the ground, and increases with height because the jet-stream winds increase with height.

$$CC \propto -\frac{\Delta E_{net}}{\Delta y} + Curv(F_{y\ wave}) + \frac{\Delta MG}{\Delta z} \tag{11.42}$$

circulation *radiation* *wave−heat* *wave−momentum*

$CC_{polar} \propto$ positive + positive + positive = **positive**

$CC_{midlat} \propto$ positive + negative + negative = **negative**

$CC_{tropics} \propto$ positive + positive + positive = **positive**

Thus, horizontal planetary waves at midlatitudes are so effective that they reverse the vertical circulation.

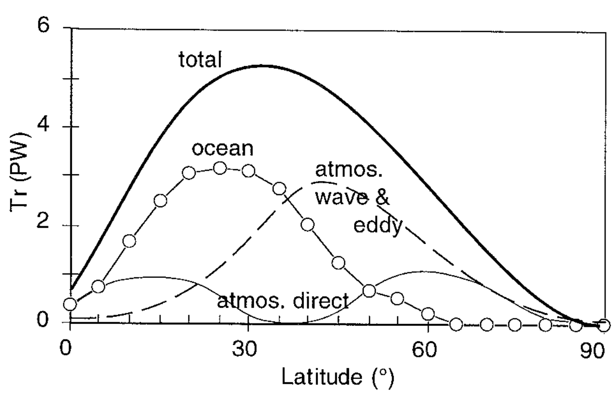


Figure 11.29
Northward transport of heat. (1 PW = 1 petawatt = 10^{15} W). At each latitude, the three thin curves sum to equal the total (thick) curve.

Earlier in this chapter, the differential heating (Fig 11.6) was summed over all latitudes (eq. 11.9) to give the net transport needed by various fluid circulations to compensate the radiative imbalance. The curve that resulted (see the solved problem in that section) is reproduced in Fig 11.29 for just the N. Hemisphere, and is labeled "total".

Observations of the earth/atmosphere/ocean system suggest how the various circulations contribute toward the total circulation (Fig 11.29). As we saw from eq. (11.42), the planetary wave circulation (and its associated high and low pressure eddies) dominate at mid latitudes, where the indirect circulation is called a **Ferrel cell**. Vertically direct circulations dominate elsewhere. The oceans also play a major role.

EKMAN SPIRAL IN THE OCEAN

As the wind blows over the oceans it drags along some of the water. The resulting ocean currents turn under the influence of Coriolis force (Fig 11.30), eventually reaching an equilibrium given by:

$$U = \left[\frac{u_{*water}^{\ 2}}{(K \cdot f_c)^{1/2}}\right] \cdot \left[e^{\gamma \cdot z} \cdot \cos\left(\gamma \cdot z - \frac{\pi}{4}\right)\right] \tag{11.43a}$$

$$V = \left[\frac{u_{*water}^{\ 2}}{(K \cdot f_c)^{1/2}}\right] \cdot \left[e^{\gamma \cdot z} \cdot \sin\left(\gamma \cdot z - \frac{\pi}{4}\right)\right] \tag{11.43b}$$

where the x-axis and U-current direction point in the direction of the surface wind. When these current vectors are plotted vs. depth (z is positive upward), the result is a spiral called the Ekman spiral.

The friction velocity u_{*water} for water is

$$u_{*water}^{\ 2} = \frac{\rho_{air}}{\rho_{water}} \cdot u_{*air}^{\ 2} \tag{11.44}$$

where the friction velocity for air was given in Chapter 4. The depth parameter is

$$\gamma = \sqrt{\frac{f_c}{2 \cdot K}} \tag{11.45}$$

where K is the eddy viscosity (Appendix H) in the ocean. The inverse of γ is called the **Ekman layer depth**. Transport of water by Ekman processes is discussed further in the Hurricane chapter.

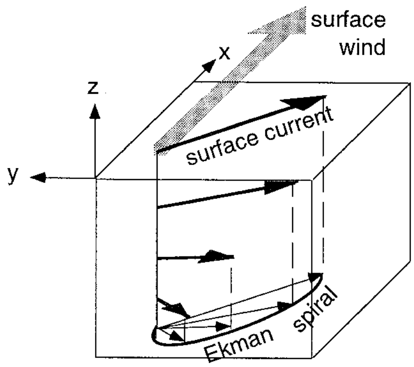
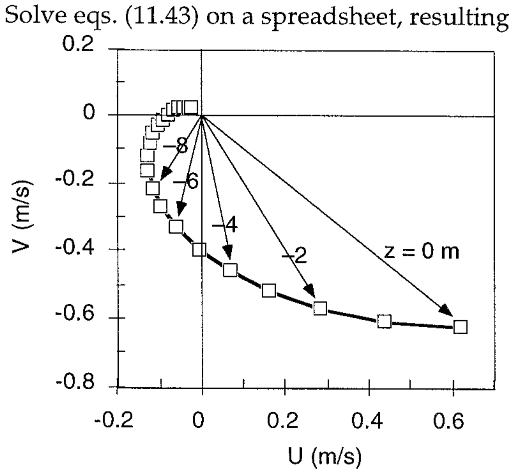


Figure 11.30
Ekman spiral of ocean currents.

Solved Example (§)
Plot the Ekman spiral in the ocean at 45° latitude, using an eddy viscosity of $2\text{x}10^{-3}$ $m^2 \cdot s^{-1}$, and $u_{*water}{}^2 = 4\text{x}10^{-4}$ m^2/s^2.

Solution
Given: $u_{*water}{}^2 = 4\text{x}10^{-4}$ m^2/s^2, $K = 0.002$ $m^2 \cdot s^{-1}$, $\phi = 45°$
Find: U & V (m/s) vs. z (m)

First: $f_c = 0.0001031$
Use eq. (11.45):
$\gamma = 0.1605$ m^{-1}
Solve eqs. (11.43) on a spreadsheet, resulting in:



Check: Units OK. Physics OK.
Discussion: The surface current is 45° to the right of the wind direction. Deeper currents decrease in speed and turn to the right due to Coriolis force. The mean transport is in the $-V$ direction.

SUMMARY

Radiation causes net heating of the tropics and cooling at the poles. The global circulation moves the excess heat from the equator to the poles to partially reduce the differential temperature.

The reduction of meridional temperature gradient is not complete, resulting in a jet stream at mid latitudes due to Coriolis force and the thermal wind effect. This jet stream is unstable, and usually meanders north and south in the form of planetary waves, also known as Rossby waves. Vorticity is defined to help explain these waves.

The waves transport so much heat and angular momentum in mid latitudes that a reverse circulation develops there, called the Ferrel cell. Elsewhere, wave and eddy circulations are weaker, allowing direct vertical circulations such as the Hadley cell in the tropics. A weak direct cell also exists in polar regions.

Atmospheric winds drive ocean currents. These currents together with the atmospheric winds transport sufficient heat from the tropics to the poles to completely counteract the radiative imbalance, putting the earth in radiative equilibrium.

Threads

Radiation (Chapt. 2) causes the differential heating that drives the general circulation. Most of this circulation is confined to the troposphere, because of the vertical temperature structure (Chapt. 1). This circulation moves both sensible and latent heat (Chapt. 3).

The jet stream, caused by various dynamical forces (Chapt. 9) meanders north and south creating troughs and ridges. This pattern creates surface high pressure areas and air masses (Chapt. 12), and low pressure areas called cyclones (Chapt. 13). Upward motion in troughs and lows causes moist (Chapt. 5) and dry (Chapt 3) adiabatic cooling (Chapt. 6), which creates clouds (Chapt. 7) and precipitation (Chapt. 8). Downward motion in ridges and highs creates relatively shallow boundary layers (Chapt. 4) that can trap air pollutants (Chapt. 17) and cause air stagnation.

Numerical models (Chapt. 14) of the general circulation are called global climate models (GCMs) when used to forecast climate change (Chapt. 18). The background environment of the general circulation governs the likely areas for hurricane (Chapt. 16) and thunderstorm formation (Chapt. 15). The Ekman spiral process causes sea level to rise ahead of landfalling hurricanes (Chapt. 16), contributing to the destructive storm surge.

EXERCISES

Numerical Problems

N1(§) Plot the idealized temperature and meridional temperature gradient at the following heights (km) above ground.

a. 1	b. 2	c. 4	d. 6
e. 8	f. 10	g. 12	h. 14

N2. Find the radiation in, out and net at latitudes

a. 10°	b. 30°	c. 45°	d. 60°
e. 15°	f. 75°	g. 5°	h. 80°

N3. Find the latitude-band accumulated radiation in and out at latitudes:

a. 10°	b. 30°	c. 45°	d. 60°
e. 15°	f. 75°	g. 5°	h. 80°

N4. Find the latitude-accumulated differential heating at latitudes:

a. 10°	b. 30°	c. 45°	d. 60°
e. 15°	f. 75°	g. 5°	h. 80°

N5. Check to see if the data in Fig 11.6 does give zero net radiation when averaged from pole to pole.

N6. What are the thermal wind vector components at 45°N for a thickness (m) of the 100-85 kPa layer that decreases northward by the amount given below per 1000 km horizontal distance?

a. 50	b. 100	c. 150	d. 200
e. 250	f. 300	g. 350	h. 400

N7. a. Using the idealized surface temperature at the equator and at the pole, find the thermal wind (gradient of geostrophic wind) based on this temperature difference.

b. Same as (a) but at 11 km agl.

c. Same as (a) but at 15 km agl.

N8. Given a Coriolis parameter of 10^{-4} s^{-1}, average temperature of 10°C, and temperature decreasing 10°C toward the east across a distance of 500 km, find the geostrophic wind at the following heights (km) above ground.

a. 1	b. 2	c. 4	d. 6
e. 8	f. 10	g. 12	h. 14

N9(§). Plot the geostrophic (jet) stream speed vs. latitude using eq. (11.14), for these altitudes (km):

a. 1	b. 2	c. 4	d. 6
e. 8	f. 10	g. 12	h. 14

N10. (§) Plot the vertical profile of baroclinic jet-stream speed at latitudes:

a. 20°	b. 30°	c. 40°	d. 60°
e. 15°	f. 75°	g. 5°	h. 80°

N11. Considering conservation of angular momentum, an air parcel from 45° would have what speed relative to earth at destination latitude:

a. 0°	b. 30°	c. 40°	d. 60°
e. 15°	f. 75°	g. 5°	h. 80°

N12. Given the following wind shears (m/s) across 100 km, calculate the relative vorticity.

a. 1	b. 2	c. 3	d. 5
e. 8	f. 10	g. 15	h. 20

N13. Given a wind of 5 m/s rotating clockwise around a high-pressure center, find the relative vorticity at the following radii (km):

a. 50	b. 100	c. 150	d. 200
e. 300	f. 500	g. 800	h. 1000

N14. For the previous two problems, find the absolute vorticity if the flow is located at latitude of 60°N.

N15. If the flow is 11 km thick, find the potential vorticity for the previous problem.

N16(§). Using the shear data from Fig 11.12 for pure zonal flow, plot the following vs. latitude:

a. relative vorticity b. absolute vorticity

N17. Suppose water in your sink is in solid body rotation at one revolution per 5 seconds, and your sink is 1 m in diameter and 0.5 m deep.

a. Find the potential vorticity.

b. If you suddenly pull the stopper and the fluid stretches to depth 1 m in your drain, what is the new relative vorticity and new rotation rate?

N18. If fluid having no relative vorticity at 45°N moves to latitude ___N, find its new relative vorticity.

a. 0°	b. 30°	c. 40°	d. 60°
e. 15°	f. 75°	g. 5°	h. 80°

N19(§). Plot the meridional displacement of a barotropic wave of wavelength 400 km centered at latitude 45°, if imbedded in background flow (m/s) given below. Assume A = 100 km.

a. 10	b. 20	c. 30	d. 50
e. 60	f. 100	g. 150	h. 200

N20. Find the barotropic beta parameter at latitudes

a. 0°	b. 30°	c. 40°	d. 60°
e. 15°	f. 75°	g. 5°	h. 80°

N21. Suppose zonal flow at latitude 45° initially has no relative vorticity, but then flows over a mountain such that its depth changes from 11 km to a depth (km) given below. Find its new relative vorticity. Which way will the wind turn?

a. 5	b. 6	c. 7	d. 8
e. 9	f. 10	g. 12	h. 13

N22. Find the Rossby radius of deformation for an 11 km deep troposphere in the standard atmosphere at latitudes:

a. 20°	b. 30°	c. 40°	d. 45°
e. 50°	f. 55°	g. 60°	h. 70°

N23. Find the dominant wavelength of baroclinic waves for the previous problem.

N24. Find the phase speed for baroclinic waves of the previous problem.

N25(§). Plot the meridional displacement of a baroclinic wave given λ = 4000 km, z_T = 11 km, and A = 500 km. The wave is centered at 45° latitude, and is in a standard atmosphere. The intrinsic phase speed (m/s) is:

a. –3	b. –5	c. –7	d. –10
e. –12	f. –15	g. –2	h. –8

N26(§). For the previous problem, find amplitudes of:

a. meridional displacement
b. vertical displacement
c. potential temperature
d. pressure
e. U-wind
f. V-wind
g. W-wind

N27(§). For the previous problem, plot separate vertical cross sections showing perturbation values for those variables.

N28. Measure the circulation of the Hadley cell, if the updraft and downdraft velocities are 0.1 m/s, and the meridional velocities (m/s) are given below. Assume the Hadley cell fills the troposphere between 0° and 30° latitude.

a. 3	b. 5	c. 7	d. 10
e. 12	f. 15	g. 20	h. 25

N29(§). Plot the Ekman spiral at 15°N if the the wind in a neutral boundary layer is blowing from the ENE at 10 m/s, for eddy viscosities (m^2/s) of:

a. 0.001	b. 0.002	c. 0.003	d. 0.004
e. 0.005	f. 0.006	g. 0.007	h. 0.008

Understanding & Critical Evaluation

U1. Express the 5 PW of heat transport typically observed at 30° latitude in other units, such as

a. horsepower   b. megatons of TNT

Hint, 1 **Megaton of TNT** $\cong 4.2 \times 10^{15}$ J.

U2. Although angular momentum conservation is not a good explanation for the jet stream, can it explain the trade winds? Discuss with justification.

U3(§). Combine eqs. (11.1) and (11.3), consider adiabatic cooling with altitude to modify the result, in order to plot the air temperature at 15 km altitude vs. latitude.

U4(§). Combine eqs. (11.1) and (11.4) to reproduce the "out" curve of Fig 11.5.

U5(§). On a spreadsheet, reproduce the results of the solved example calculating E_{in} and E_{out} at all latitudes (taking 5° increments). Use this spreadsheet to calculate and reproduce Fig 11.6 on the differential heating. Also use this to produce the curve of transport Tr vs. latitude.

U6. Derive the equation for jet stream wind speed (11.14). Describe the physical meaning of each term in that equation. Also, describe the limitations of that equation.

U7. a. What is the sign of CC for a direct circulation in the S. Hemisphere?

b. Determine the signs of terms in eq. (11.42) for the S. Hemisphere.

U8. If tangential velocity increases as you cross the wall of a tornado from outside to inside, how does relative vorticity change with radius R from the center of the tornado?

U9. Compare the barotropic and baroclinic relationships for phase speed of planetary waves. Which is fastest (and in what direction) in mid-latitudes?

U10. If the troposphere were isothermal everywhere, find the number of planetary waves that would encircle the earth at 45°N, using baroclinic theory. How does this differ from barotropic theory?

U11. Discuss the differences between barotropic and baroclinic theories and their underlying assumptions for planetary waves.

U12. Using eq. (11.30) for the north-south displacement of a baroclinic wave, discuss (and/or plot), how the location of the crest of the wave changes with altitude z within the troposphere.

U13. How does static stability affect the phase speed of baroclinic planetary waves?

U14. Where with respect to the ridges and troughs in a baroclinic wave would you expect to find the greatest: (a) vertical displacement; (b) vertical velocity; (c) potential temperature perturbation?

U15. Why does the expression for cell circulation contain the Coriolis parameter and Brunt-Väisälä frequency?

U16. Would there be an Ekman spiral in the ocean if there was no Coriolis force? Explain.

Web-Enhanced Questions

W1. Download a northern or southern hemispheric map of the current winds (or pressures or heights from which winds can be inferred), and identify regions of near zonal flow, near meridional flow, extratropical cyclones, and tropical cyclones (if any).

W2. Download an animated loop of geostationary satellite photos (IR or water vapor) for the whole earth disk, and identify regions of near zonal flow, near meridional flow, extratropical cyclones, and tropical cyclones (if any).

W3. Download a still, daytime, visible satellite image for the whole disk from a geostationary satellite, and estimate fractional cloudiness over the different latitude belts. Use this to estimate the incoming solar radiation reaching the ground in each of those belts, and plot the zonally-averaged results vs. latitude.

W4. Download an IR satellite image showing the entire earth (i.e., a whole disk image), and compute the effective radiation temperature averaged around latitude belts. Use this with the Stephan-Boltzmann equation to estimate and plot E_{out} vs. latitude due to IR radiation.

W5. Download rawinsonde data from a string of stations that cross through the center of a low pressure center. Create a vertical slice through the atmosphere similar to Fig 11.3a, and confirm that the point of low pressure on a constant height line corresponds to the point of low height of an isobar.

W6. Download weather maps of temperature for the surface, and for either the 85 kPa or 70 kPa isobaric surfaces. Over your location, or other location specified by your instructor, use the thermal wind relationship to find the change of geostrophic wind with height at the surface, and at the isobaric level you chose above the surface.

W7. Download a weather map for the 100-50 kPa thickness, and calculate the components of the thermal wind vector for that surface. Also, what is the thermal wind magnitude? Draw arrows on the thickness chart representing the thermal wind vectors.

W8. Download data from a string of rawinsonde stations arranged north to south that cross the jet stream. Use the temperature, pressure, and wind speed data to produce vertical cross-section analyses of: (a) temperature; (b) pressure; and (c) wind speed. Discuss how these are related to the dynamics of the jet stream.

W9. Download weather maps of height contours for the 20 or 30 kPa isobaric surface, and draw a line within the region of most closely-spaced contours to indicate the location of the jet stream core. Compare this line to weather maps you can download from various TV and weather networks showing the jet stream location. Comment on both the location and the width of the jet stream.

W10. Download a weather forecast map that shows vorticity. What type of vorticity is it (relative, absolute, potential, isentropic)? What is the relationship between vorticity centers and fronts? What is the relationship between vorticity centers and bad weather (precipitation)?

W11. Download surface weather observations for many sites around your location (or alternately, download a weather map showing wind speed and direction), and calculate the following vorticities at your location: (a) relative; (b) absolute; (c) potential.

W12. Download a map of height contours for either the 50, 30, or 20 kPa surface. Measure the wavelength of a dominant planetary wave near your region, and calculate the theoretical phase speed of both barotropic and baroclinic waves for that wavelength. Compare and discuss.

W13. Download maps at a variety of altitudes and for a variety of fields (e.g., heights, temperature, vertical velocity, etc.), but all valid at the same time, and discuss how they are related to each other using baroclinic theory.

W14. View an animation of a loop of IR or water vapor satellite images for the whole disk of the earth. Identify the three bands of the general circulation, and discuss how they deviate from the idealized picture sketched in this chapter. To hand a result in, print one of the frames from the satellite loop and sketch on it lines that demark the three zones. Speculate on how these zones vary with season.

W15. Download a map of recent ocean surface currents, and compare with a map showing the general circulation winds for the same time. Discuss how the two maps are related.

Synthesis Questions

S1. If the earth did not rotate, how would the steady-state general circulation be different, if at all?

S2. If our current rotating earth with current general circulation suddenly stopped rotating, describe the evolution of the general circulation over time. This is called the **spin-down** problem.

S3. If the earth rotated twice as fast, how would the steady-state general circulation be different, if at all?

S4. If our current rotating earth with current general circulation suddenly started rotating twice as fast, describe the evolution of the general circulation over time. This is a **spin-up** situation.

S5. If radiative heating was uniform between the equator and the poles, how would the steady-state general circulation be different, if at all?

S6. If the equator were snow covered and the poles were not, how would the general circulation change, if at all?

S7. If an ice age caused the polar ice caps to expand to 50° latitude in both hemisphere, describe how general circulation would change, if at all?

S8. Suppose the troposphere were half as deep. How would the general circulation be different, if at all?

S9. What if potential vorticity was not conserved. How would the general circulation be different, if at all?

S10. Suppose the average zonal winds were faster than the phase speed of short waves, but slower than the phase speed for long waves. Describe how the weather and weather forecasting would be different at midlatitudes, if at all?

S11. Suppose that short waves moved faster than long waves, how would the weather at midlatitudes be different, if at all?

S12. Suppose the troposphere was statically unstable everywhere. Could baroclinic waves exist? If so, how would they behave, and how would the weather and general circulation be different, if at all?

S13. If planetary waves did not transport momentum, heat, and moisture meridionally, how would the weather and climate be different, if at all?

S14. If the ocean currents did not transport any heat, how would the atmospheric general circulation and weather be different, if at all?

S15. What if the ocean surface were so slippery that there would be zero wind drag. Would any ocean currents develop? How? Why?

Second Edition

Meteorology *for* Scientists and Engineers

Roland B. Stull