6 Estimation of the Fractal Dimension of Terrain from Lake Size Distributions

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INTRODUCTION

Examination of the scaling of geomorphological features yields information on the processes that shaped the terrain (Church and Mark, 1980; Milne, 1988). In floodplains, which are often a lake-rich terrain, the lakes reflect the pattern of depressions and the overall relief. The sizes and shapes of floodplain lakes, which can be measured readily by remote sensing, reveal small differences in relief that do not appear on conventional topographic maps. The origin of some kinds of floodplain lakes is apparent from their size and shape. Oxbow lakes and scroll lakes, which are created by migrating river channels (Hutchinson, 1957), are two such examples. Many floodplain lakes are irregular in shape, however, and the geomorphological processes that formed the lakes are less apparent. Comparison of scaling patterns between these different kinds of floodplain lakes could provide insight into the processes that shape the floodplain (Salo, 1990).

The frequency distribution of lake sizes can be used in analyzing the scaling of lake morphometry. Such an analysis is based upon a comparison of the distribution of empirical observations with that which would be expected in the absence of a predominant scale-specific geomorphological process (Goodchild, 1981). Fractal theory provides a stochastic model of scale-invariant relief that is useful as a point of reference or 'null hypothesis' for this comparison. The relation between area and perimeter in a population of lakes also provides information on scaling and can be analyzed for consistency with the stochastic model (Mandelbrot, 1982).

In this study, we analyze the statistical properties of size distributions for floodplain lakes of the Amazon and Orinoco rivers. For the Orinoco floodplain lakes, we also

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examine the relation between area and perimeter. Our objectives are to evaluate the consistency of these statistical properties with the fractal model, and to explore the usefulness of lake size distributions as indicators of geomorphological patterns and processes on floodplains and other kinds of lake-rich terrain.

CONCEPTUAL BACKGROUND

Mandelbrot (1977, 1982) introduced the term 'fractal' for a spatial or temporal phenomenon whose variation is scale-invariant. The variation in a fractal phenomenon can be described mathematically by a non-integer dimension (the fractal dimension, D). Many geophysical phenomena, including coastlines, topography, island areas, river discharges, and climatic variation, show fractal characteristics over a wide range of observational scales (Mandelbrot, 1982; Burrough, 1985; Kaye, 1989; Takayasu, 1990). Computer simulations of fractal surfaces based on a stochastic process called fractional Brownian motion (FBM) produce images that resemble closely certain types of real terrain (Mandelbrot, 1982; Goodchild, 1988). This resemblance is surprising given the scale-specific nature of most geomorphological processes, and suggests that the types of terrain that are simulated realistically by FBM models are shaped primarily by geomorphological processes that are scale-invariant, and that the terrain can be described by the same fractal dimension that characterizes FBM surfaces.

Mandelbrot used conceptual models to demonstrate that islands created by flooding a fractal landscape have a size distribution that reflects the fractal geometry of the landscape. The statistical properties of the island area–number relationship had been studied earlier by Korčak (1940), who found that an inverse power law fitted empirical data on the size distribution of oceanic islands. If the islands are ranked by area from largest to smallest, then the number of islands (A) above a given size (a) can be denoted as $Nr(A > a)$ and is given by:

$$Nr(A > a) = F'a^{-B}$$

where F' and B are positive constants. Mandelbrot called this the Korčak Empirical Law, and he argued that the exponent B should be equal to $D/2$, where D is the fractal dimension of the relief (i.e. of the topographic contours). Fractal dimension D lies between 1 (Euclidean shapes with smooth boundaries) and 2 (infinitely convoluted boundaries). The graphical diagnosis for goodness-of-fit to the Korčak Law is a log–log plot of rank against area, which will yield a straight line of slope $-B$. This plot is called a Pareto fit; it takes the same form as an asymptotically hyperbolic probability distribution called the Pareto distribution, which describes diverse socioeconomic and biological phenomena and is known particularly for its application in economics for modeling the upper tail of the distribution of income (Badger, 1980; Arnold, 1985; West and Shlesinger, 1990).

Lake areas also reportedly follow the Korčak Law (Mandelbrot, 1982), as noted originally by Korčak (1940), although we have found few empirical demonstrations of this fit in the literature (notable exceptions include Goodchild (1981) and Kent and Wong (1982) who studied lakes on glacial landscapes). Mandelbrot speculated that landscapes with fractal relief will have lake area distributions that follow the Korčak

Law, and that the fractal dimension D therefore can be estimated from B, the exponent of the lake size distribution. Although a multitude of factors in addition to relief may affect the occurrence and size of lakes on a fractal landscape, Mandelbrot hypothesized that these factors may act independently of lake size, with little resultant effect on the original hyperbolic size distribution described by the Korčak Law.

An independent method to determine the fractal dimension of lake-rich terrain is based on the relation between area and perimeter in a population of lakes of varying size that is observed at a single scale of observation (Mandelbrot, 1982; Burrough, 1985). For a population of lakes with smooth boundaries ($D = 1$), the perimeter will vary as area to the one-half power. The perimeter of lakes on fractal surfaces should vary as area to the power $D/2$, because the larger lakes appear to have more convoluted boundaries at a fixed observational scale. The slope of a log–log plot of perimeter against area is constrained to lie between 0.5 (Euclidean shapes with $D = 1.0$) and 1.0 (infinitely convoluted boundaries with $D = 2.0$). Theoretically, the estimate of D obtained in this way will agree with that obtained from the Korčak Law. However, only a few studies have estimated D by both methods. Goodchild (1981) reported disagreement between the two estimates for lakes on Random Island, off the coast of Newfoundland, while Kent and Wong (1982) found reasonable agreement for Canadian Shield lakes in Ontario. The two methods yielded different estimates of D for lakes simulated on FBM surfaces; the Pareto fit produced an infeasible estimate of D (Goodchild, 1988).

The fractal model has been proposed as a point of reference or 'null hypothesis' for the analysis of lake-rich landscapes: comparison of the statistical properties of empirical lake size distributions with those predicted by the FBM model might reveal the presence or absence of scale-specific geomorphological processes that shape the relief and thereby determine patterns in lake morphometry (Goodchild, 1988). If the empirical lake size distribution possesses distinct statistical properties, then we can reject the FBM model and consider particular scale-specific processes that might be responsible for the genesis of the lakes. It should be noted that we use the term 'null hypothesis' only in an informal sense; we are not referring to a statistical test based on probability theory.

STUDY SITES

Amazon Floodplain

The Amazon River, which is the world's largest river, has a mean discharge of ca. 210 000 $m^3\ s^{-1}$ (Richey *et al.*, 1989) and inundates a floodplain of 170 000 km^2, of which 65% is located within Brazil (Melack and Fisher, 1990). We will limit this description to the Brazilian Amazon (Figure 6.1), which is the study site analyzed in this paper. The main stem of the Amazon in Brazil is called the Solimões River above the confluence with the Negro River at Manaus. Along the 2600 km of channel from the Xingú River confluence up-river to the Peruvian border, the floodplain is typically 10–50 km wide and is composed of a mosaic of open waterbodies, dense fields of herbaceous plants that float on the water surface during floods, and forest. At Manaus on the central Amazon, the river stage varies seasonally by ca. 10 m. Many of the

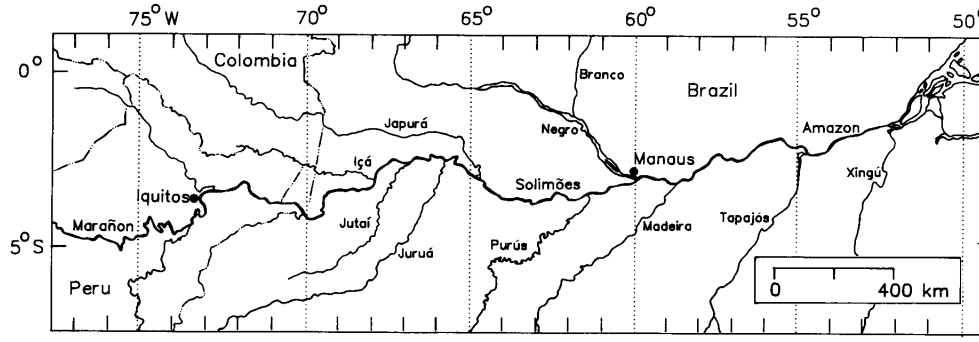

Figure 6.1 Map of the Amazon River

largest lakes are connected with the river throughout the year, and their water levels are controlled directly by the river.

Most of the suspended sediment carried by the Amazon main stem originates in the Andes along the western margin of the basin, and enters the Brazilian Amazon through the main stem and through the Madeira River (Meade *et al.*, 1985). Tributaries that drain lowland basins, including the Negro and Tapajós rivers, carry relatively little material in suspension, and their waters have a black or clear appearance.

Lake outlines for two distinct reaches of the Amazon floodplain are depicted in Figure 6.2. In the upper panel, which is near the confluence with the Japurá River, the floodplain is dominated by scroll-bar topography and contains numerous small, narrow lakes (channel lakes). Channel lakes on the main stem Amazon floodplain occur principally in association with scroll bars; few channel lakes have dimensions that approach those of the main river channel. Mertes (1985) noted that the curvature of meander scrolls on the Amazon floodplain tends to resemble that of small channels (paranás) that carry water from the main channel across the levees and into the floodplain, as opposed to that of the meanders of the main channel. Mertes concluded that the paranás probably create most of the meander scrolls.

The lower panel of Figure 6.2 depicts the floodplain near Manaus, which has both scroll-bar topography and back-swamp deposits with round or irregular lakes. Also, back-flooded tributary valleys (blocked-valley lakes) form dendritic lakes along the floodplain boundary.

The origin of blocked-valley lakes has attracted attention from various authors, and several hypotheses have been advanced. One view holds that eustatic changes in sea level during the Flandrian transgression caused changes in river base level, backing up the Amazon main stem and flooding lateral tributary valleys. The backwater effect resulted in enhanced aggradation of floodplains inundated by turbid river water. Where less turbid influents entered the floodplain of a turbid river, the flooded valleys were not filled at comparable rates and persist today as blocked-valley lakes (Tricart, 1977; Klammer, 1984). Other investigators have provided geomorphological evidence for neotectonic sinking of lower river courses along fault lines, resulting in box-shaped valleys that are flooded permanently (Departamento Nacional da Produção Mineral, 1978). Damming of tributary valleys by fluvial deposition from the main stem also has

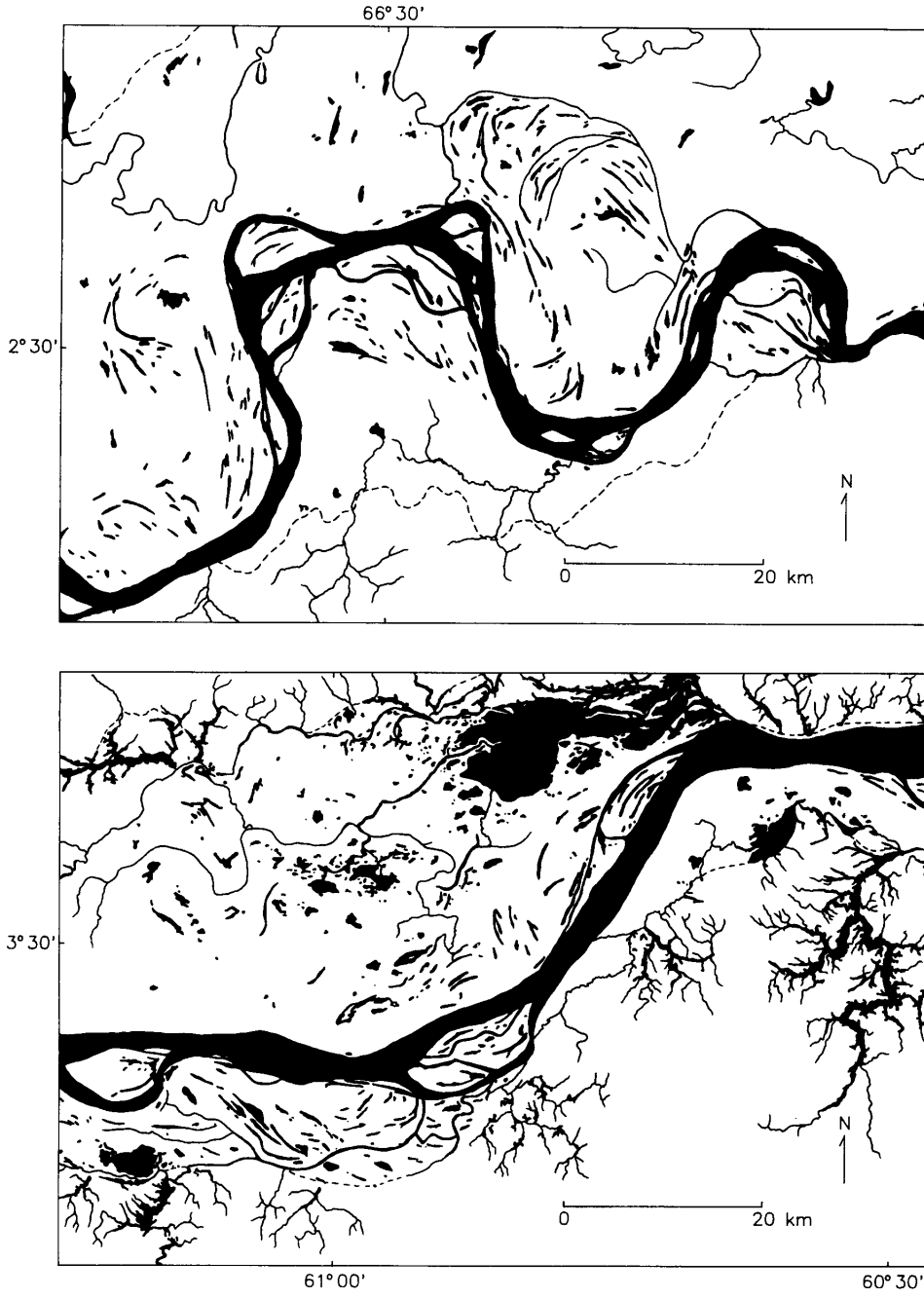

Figure 6.2 Examples of the Amazon floodplain. Solid areas indicate open-water surfaces, and dashed lines show the floodplain boundaries. The upper panel is a reach near the confluence of the Japurá River, and the lower panel is a reach near Manaus. (Adapted from planimetric maps published by Departamento Nacional da Produção Mineral, 1978)

been suggested (Iriondo, 1982). These processes may operate separately or in combination to produce the blocked-valley lakes along the Amazon (Iriondo, 1982). Regardless of the processes that result in formation of blocked-valley lakes, the size distribution of the lakes is expected to be determined by the size distribution of the tributary stream valleys that were produced by fluvial erosion before the valleys became back-flooded.

Orinoco Floodplain

The Orinoco River of Venezuela and Colombia (Figure 6.3) is the world's third largest river in terms of discharge (mean, 36 000 $m^3 s^{-1}$: Meade *et al.*, 1983). The floodplain that fringes the main stem covers 7000 km^2 along the lower 770 km of channel between the delta and the Meta River (Hamilton and Lewis, 1990a). A more extensive savanna floodplain (the *Llanos*) occurs west of the main stem and is contiguous with the main stem floodplain between the Meta and Apure rivers. Scroll-bar topography is less common on the Orinoco floodplain than on the Amazon floodplain. The vegetation of the Orinoco floodplain resembles that of the Amazon. The level of the Orinoco River fluctuates seasonally by 10–15 m along the lower main stem. However, few lakes on the Orinoco floodplain fluctuate in level as much as the river because the lakes are usually isolated from and perched above the river during low water (Hamilton and Lewis, 1990b). Like the Amazon, the sediment load of the Orinoco is derived almost entirely from the Andes to the west, and enters the Orinoco main stem largely through the Meta and Apure Rivers.

Examples of the Orinoco floodplain are shown in Figure 6.4. The upper panel is a reach between the Meta and Apure rivers, where lakes are particularly abundant along the left bank. The lower panel shows the floodplain in a reach between the Caura and Caroní rivers, where the floodplain is interrupted periodically by outcrops of high ground. Floodplain bordering the main stem is narrower than along the Amazon, and the lakes are generally smaller. Blocked-valley lakes are rare on the Orinoco floodplain.

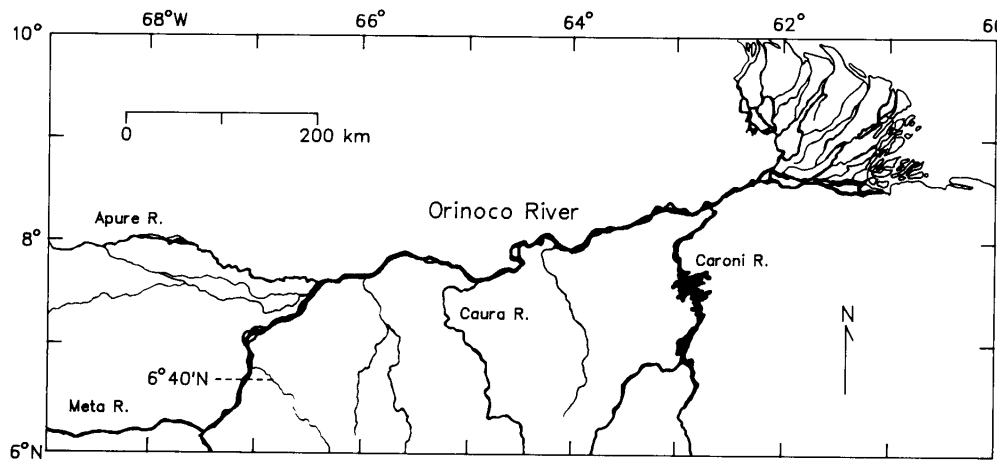

Figure 6.3 Map of the Orinoco River

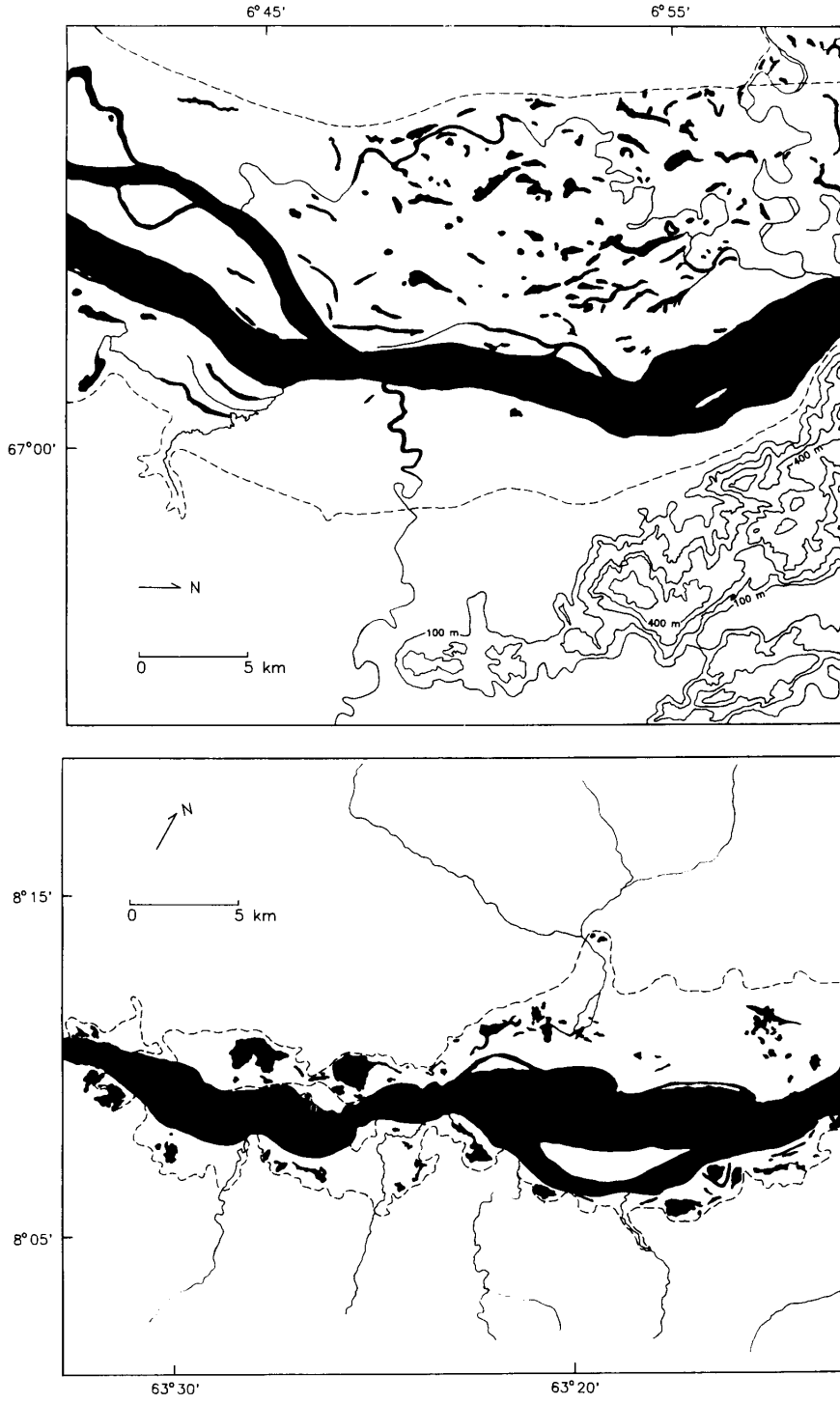

Figure 6.4 Examples of the Orinoco floodplain. Solid areas indicate open-water surfaces, and dashed lines show the floodplain boundaries. The upper panel is a reach between the Apure and Meta rivers, and the lower panel is a reach between the Caura and Caroní rivers. (Adapted from maps published by the Venezuelan Dirección de Cartografía Nacional)

METHODS

Amazon Lake Measurements

Until recently, analysis of the morphometric characteristics of Amazon floodplain lakes was difficult because of the lack of appropriate imagery and maps for the floodplain, much of which is remote and often obscured by cloud cover. However, side-looking airborne radar (SLAR, X-band) imagery, published by the Brazilian *Projeto RadamBrasil* (e.g. Departamento Nacional da Produção Mineral, 1978), covers the entire Brazilian Amazon Basin. This radar imagery provided the opportunity to measure lake sizes and shapes on the Amazon floodplain because of the high contrast between open water surfaces, which cause specular reflection of microwave radiation, and soil and vegetation, which backscatter part of the incident radiation.

We used a combination of prints of semi-controlled radar mosaics and the planimetric maps that were produced from these images (both at 1 : 250 000 scale) to identify and measure lakes. The planimetric maps were used in a few cases for which the print quality of the mosaic was poor, and for areal measurements of the larger lakes.

We measured all lakes (N = 6510) on the fringing floodplain of the main stem Amazon River system in Brazil (Figure 6.1). We define the fringing floodplain as area that is inundated seasonally by water from the parent rivers, following Welcomme (1979). Our study area extends from the apex of the Amazon delta up-river to the Peruvian border. Our data include measurements of the shape and length reported by Melack (1984), except that we have excluded lakes of the major tributaries. We have expanded the data set to include lake area as well as shape and length.

Lake measurements included length and width, and area was either measured or estimated. The maximum length (longest straight line across the lake without crossing land) and maximum width (perpendicular to its length) of each lake were measured with a ruler. The areas of the largest 10–15% of the lakes and of all dendritic lakes were measured by electronic planimetry. These lakes appear to be the most irregular in shape. The remaining lakes appear to be elliptic in shape, and their areas could thus be estimated from length and width measurements and the formula for area of an ellipse. Between 5 and 10% of the lakes whose areas were estimated in this manner were irregular, crescentic, or sinuous rather than elliptic in shape; we have probably underestimated their areas. Our measurements were less accurate for the smallest lakes. Lakes with a maximum length $\leqslant$ 250 m ($\leqslant$ 1.0 mm at a scale of 1 : 250 000) were difficult to distinguish on the images.

Orinoco Lake Measurements

The Orinoco lakes were measured on maps of 1 : 100 000 scale produced from aerial photography by the Venezuelan Dirección de Cartografía Nacional between 1968 and 1975. In the course of field studies on the Orinoco floodplain, we have confirmed that lake and river boundaries on the maps generally correspond to the vegetated shorelines, and hence can be considered the low-water boundaries (i.e. areas that are flooded most of the year).

We measured all lakes (N = 2294) on the fringing floodplain of the main stem Orinoco River from 6°40′N latitude down-river to the apex of the delta (Figure 6.3).

Maps were unavailable for a 70 km reach of floodplain that extends from 6°40′N latitude up-river (southward) to the confluence with the Meta River; this reach comprises only about 7% of the total area of fringing floodplain (Hamilton and Lewis, 1990a).

The area and perimeter of each lake were measured with a computerized digitizer. The accuracy of our measurements was lower for lakes with a maximum length $\leqslant$ 100 m ($\leqslant$ 1.0 mm at a scale of 1 : 100 000).

Definition of Floodplain Lakes

We define floodplain lakes as basins that contain water throughout the year. During inundation by the river, the floodplain is typically covered with a continuous sheet of water, within which open-water areas are visible. When the river level falls, water drains from the floodplain, and lakes persist only in depressions that are perched above the river level, or in deeper basins that remain in communication with the river. Floodplain forest and fields of herbaceous plants grow on most of the seasonally exposed land (Junk, 1983). During inundation, the upper portion of the forest canopy generally remains above the water level, and the herbaceous vegetation remains emergent or forms floating mats that rise and fall with the fluctuating water level. For this reason, the boundaries of floodplain lakes appear similar on the radar images regardless of whether the surrounding vegetation was flooded or dry. The shapes of open-water areas that appear on the Orinoco maps, which were drawn from aerial photography, also appear to represent the approximate low-water boundaries of floodplain lakes.

Although in most cases the identification of a discrete floodplain waterbody was unambiguous, there were several situations that required the use of specific criteria for definition of a floodplain lake. Many lakes are interconnected by channels. If the channel is narrow (i.e. the width of a single line on the images or maps) relative to the width of the lakes on either end, the lakes are considered separate waterbodies. Enlarged areas along floodplain channels are not considered lakes unless they are at least three times wider than the channel. Waterbodies that are confluent with the river channel are considered lakes only if the width of the confluence is less than half of the maximum length of the waterbody. Dendritic back-flooded tributary valleys are common along the Amazon floodplain. We consider these to be Amazon floodplain lakes only if they are contiguous with the fringing floodplain of the main stem. Two major tributaries of the Amazon, the Tapajós and Xingú rivers, form large lake-like waterbodies in their lower reaches (Irion, 1984); these are not included in our lake measurements.

Classification of Floodplain Lakes

We subdivided our data sets into three classes of floodplain lakes according to differences in their geomorphological origins. Blocked-valley lakes, which are common on the Amazon but not on the Orinoco, extend inland from the floodplain boundary (escarpment) and are shaped like drainage valleys. All other floodplain lakes located within the boundaries of the fringing floodplain are lateral levee lakes (Hutchinson, 1957). Within this category, we distinguish lakes with dish-shaped basins

(dish lakes) from those with channel-shaped basins (channel lakes) (Hamilton and Lewis, 1990b). Dish lakes tend to have lower rates of hydraulic through-flow during inundation, resulting in gently sloping bottoms composed of fine sediment (Blake and Ollier, 1971). Channel lakes tend to have visible currents during inundation, and often have steeper sides and bottoms composed of coarser sediment, or of compacted fine sediment. Channel lakes commonly lie in swales between concentric levees of coarse sediment (scroll bars). We have chosen a length-to-width ratio of 5 as the division between these categories, following the suggestion of Hamilton and Lewis (1990b), who examined the relation between morphometry and limnology in Orinoco floodplain lakes.

RESULTS

The frequency distributions of our measurements of floodplain lake area show strong positive skew. Two common probability distributions with long upper tails are the log-normal and Pareto. The distribution of sizes (expressed as area) for all lakes on the main stem Amazon floodplain is shown in Figure 6.5, which is a log–log plot (base 10) of rank against area (Pareto fit); a Pareto distribution will appear linear on this plot with a slope of $-B$. The parabolic shape in the lower domain suggests a log-normal fit, but the data fit the Pareto distribution better in the upper domain. The transformation from apparent log-normal to Paretian tail behavior occurs at about the 75th percentile, as shown by the box plot. Pareto fits for each lake class on the Amazon floodplain show this apparent transformation from log-normal to Paretian behavior in the upper tail of each distribution (Figure 6.6).

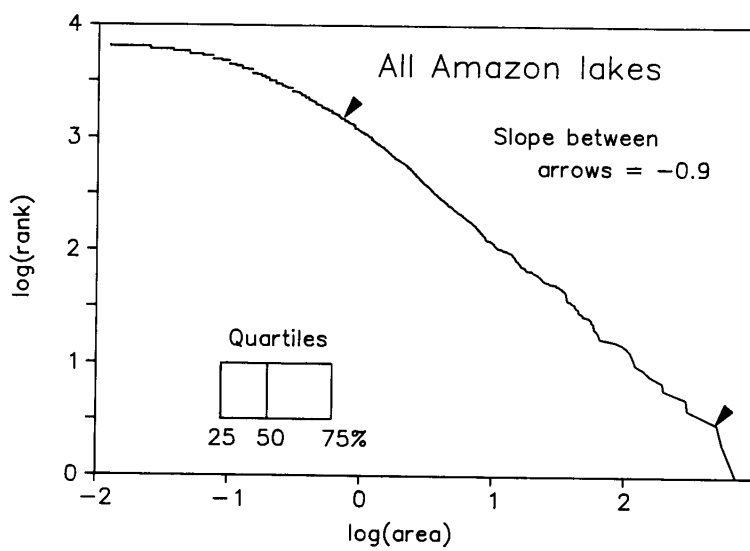

Figure 6.5 Pareto fit for all lakes on the Amazon floodplain. Linearity indicates that the size distribution of the lakes is statistically self-similar. In theory, the fractal dimension D of the surface on which the lakes occur can be estimated from the slope ($-B$) as $D = 2B$. Units of area are km^2 in this and the following plots. The box plot is aligned along the x-axis to show the quartiles of the distribution of lake areas

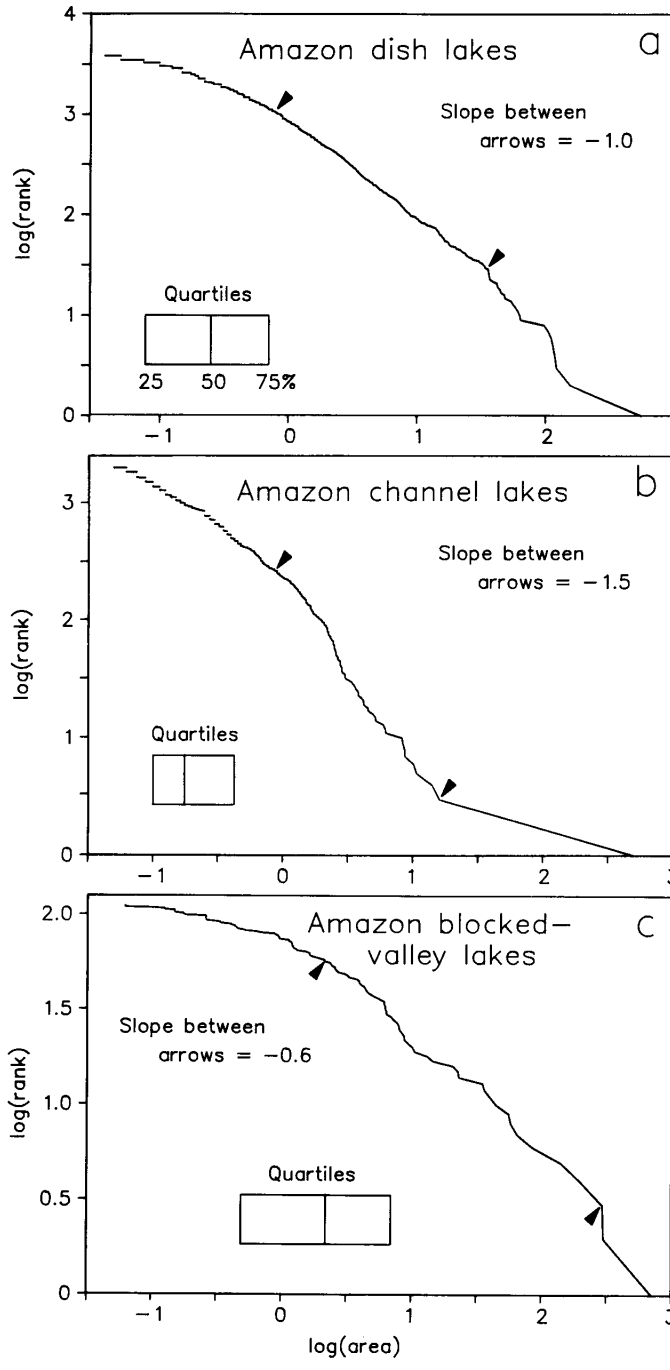

Figure 6.6 Pareto fits for dish, channel and blocked-valley lakes on the Amazon floodplain. See Figure 6.5 for explanation

The three lake classes for the Amazon floodplain show distinct slopes in the linear portion of their Pareto fits (Figure 6.6). Dish lakes, which comprise the majority (68%) of floodplain lakes that we measured along the Amazon main stem, fit the Pareto model above the 75th percentile, except that the largest lakes fall below the extrapolated Pareto line (Figure 6.6a). The slope of the linear portion of the Pareto fit for Amazon dish lakes leads to an estimate for the fractal dimension D of 2.0, which is infeasible. Channel and blocked-valley lakes also fit the Pareto model in the upper domain, although not as well as dish lakes; the values of D estimated from their slopes are 3.0 and 1.2 (Figure 6.6b). Channel lakes thus show the shortest (steepest) Paretian

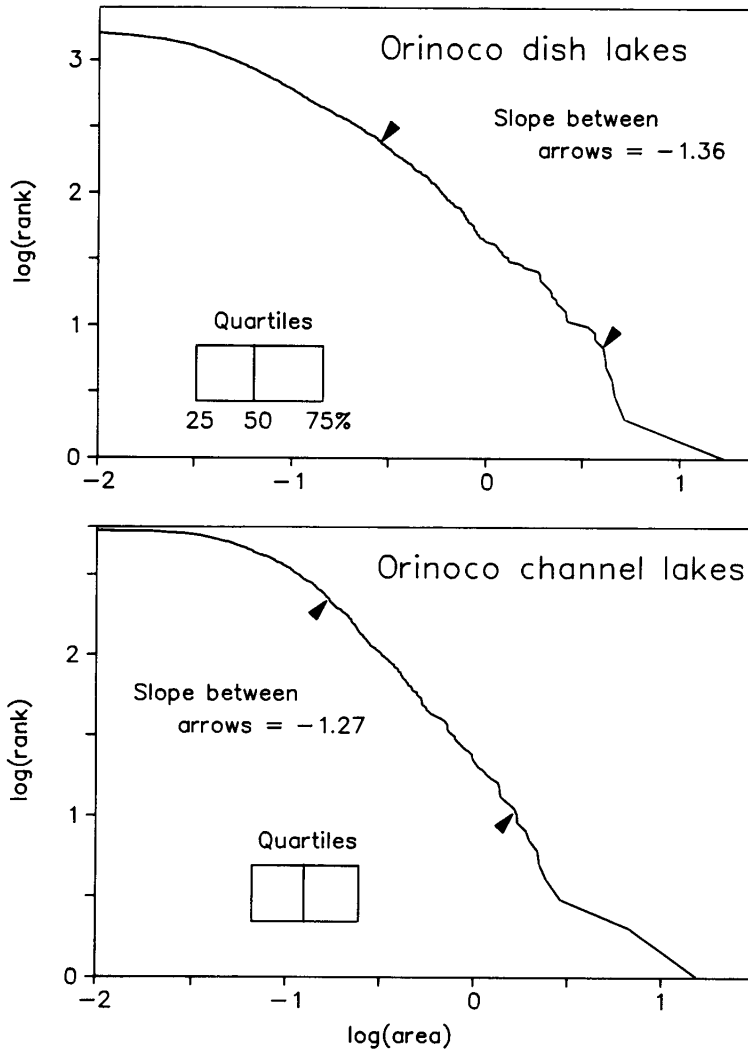

Figure 6.7 Pareto fit for dish and channel lakes on the Orinoco floodplain. See Figure 6.5 for explanation

tail, while blocked-valley lakes show the longest tail. The Pareto fit for blocked-valley lakes is the only one that yields a feasible estimate of D (1.2).

The general shapes of Pareto fits for Orinoco dish and channel lakes resemble those for the Amazon lakes (Figure 6.7). However, the range over which the fit appears linear for the Orinoco lakes lies below that of the Amazon lakes. The slopes of the Pareto fits for Orinoco dish and channel lakes in Figure 6.7 lead to infeasible estimates

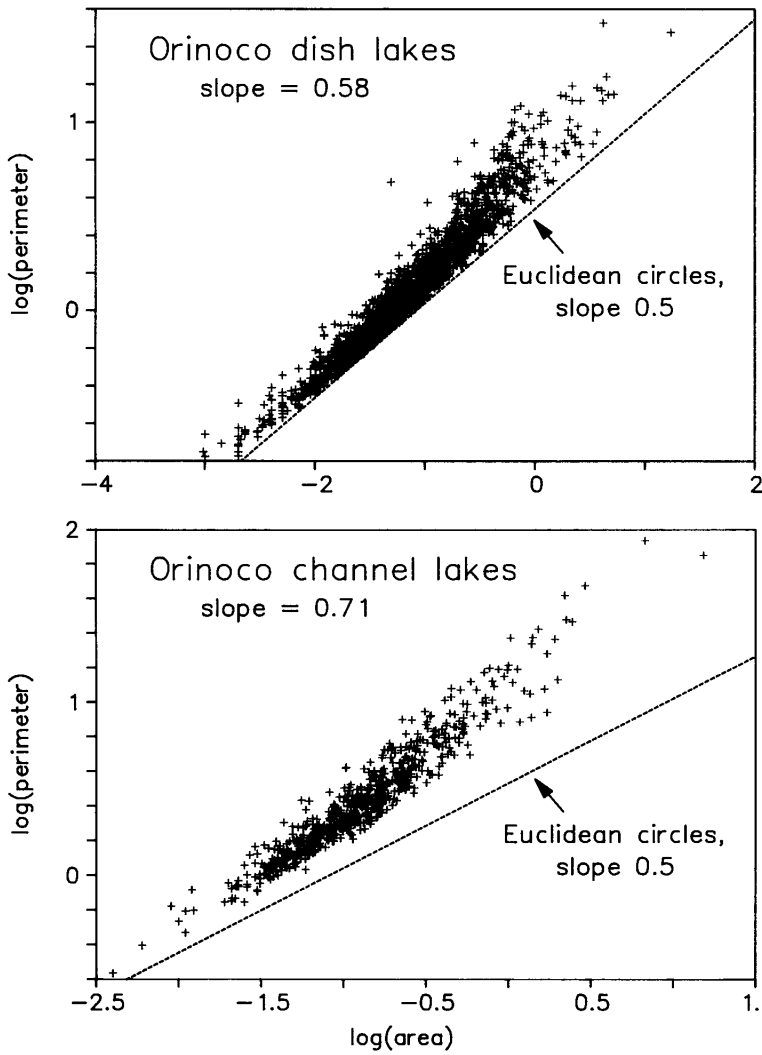

Figure 6.8 Area–perimeter relation for Orinoco dish and channel lakes. The slopes were determined by linear regression; $r^2 = 0.93$ for dish lakes and 0.92 for channel lakes. The line for Euclidean circles (smooth boundaries; $D = 1.0$) is included for comparison. In theory, the fractal dimension D of the surface on which the lakes occur can be estimated from the slope (B) as $D = 2B$; this is an independent method for estimation of the fractal dimension of terrain

of D (2.7 and 2.5), and the slopes of these two lake classes are closer to each other than to the slopes of either dish or channel lakes on the Amazon floodplain. Dish lakes outnumber channel lakes on the Orinoco floodplain by a factor of three.

Our Orinoco data set includes lake perimeters as well as areas, allowing us to examine the area–perimeter relation (Figure 6.8). Simple linear regression was used to fit lines to the empirical data in Figure 6.8. The difference between the slopes of the two regression lines is highly significant (F-test, $P < 0.001$). The slopes of regression lines lead to feasible estimates of D (1.16 for dish lakes and 1.42 for channel lakes). The difference in the area–perimeter estimates of D for the two lake classes is large compared with the similar slopes of the Pareto fits for these lakes.

DISCUSSION

Goodness-of-fit to the Pareto Distribution

Pareto fits for lake areas consistently show linearity in the upper domain, with downward departures of the data in the lower domain and on the uppermost end. This result might be interpreted as evidence for inadequacy of the Pareto distribution (Korčak Empirical Law) to describe our lake area distributions. It is also possible that the data are log-normally distributed in the lower domain, then display a transformation to Paretian behavior in the upper domain; such a transformation is observed commonly in scale-invariant phenomena, and is thought to represent a change in controlling factors (West and Shlesinger, 1990). However, in the following discussion we argue that the departures of the data at the ends can be explained and, indeed, are expected in these data, even if their true distribution is Paretian.

Departure of variates from the extrapolated fit in the tails of a probability distribution can result from truncation or censorship of the variates (Aitchison and Brown, 1963). Truncation refers to the case where values of the variate above or below a certain point either cannot occur or are not observed, whereas censorship refers to the case where, above or below a certain point, the exact values of the variates are unknown but limited knowledge of the variates is available. Both truncation and censorship appear important in the distribution of lake areas that we have presented here.

The apparent log-normal behavior in the lower domain of the distributions probably results because we approached the limits of resolution of lakes on the imagery. In the Amazon lake data (Figure 6.5), the transformation occurs at about the 75th percentile. This point corresponds to a lake area of 0.74 km^2; a circular lake of this area has a diameter of 0.5 km, which is only 2 mm on the 1 : 250 000 radar imagery used for our measurements. Lakes are likely to be increasingly difficult to discern below this size, resulting in increasing censorship of the distribution. The observation that the linear range extended further downward in the Pareto fits for the Orinoco lakes, which were measured on maps of larger scale, supports this interpretation. Coverage of lake surfaces by vegetation probably also obscures some of the smallest lakes because floating vegetation tends to extend outward from the lake shoreline; dense vegetation over water appears as land on X-band radar and in aerial photography. In addition, the smallest basins may not hold water all year simply because of

losses by evaporation and seepage. For these reasons, censorship is expected to occur for the smallest lakes even if we could measure them accurately, resulting in a downward departure of the data from the extrapolated Pareto line. Eventually a point of truncation must be reached, below which no lakes can be recognized. Truncation of the lower tail sets the minimum lake area, but unlike censorship it does not affect the slope, because the lakes are ranked from largest to smallest.

Censorship and truncation of the largest lakes also are expected because the floodplain occupies a finite space between the river channel and the escarpment. As lakes become larger, they are increasingly subject to constraints on their area as their boundaries encounter the edges of the floodplain. There are thus fewer large lakes, limited areas of the largest lakes, or both, resulting in increasing censorship of the uppermost end of the distribution. The finite area of floodplain also dictates that there must be a maximum possible lake size, which results in a point of truncation for the distribution.

Censorship and truncation of the upper tail of a Pareto distribution cause the distribution to depart from linearity because the variates are ranked from largest to smallest. A censored or truncated Pareto fit may appear to be approximately linear in the lower domain, but the slope of the 'linear' portion may be altered considerably. To demonstrate the effects of censorship and truncation on the slopes of the Pareto fits, we created an idealized Pareto distribution of lake areas with slope = -0.6 (D = 1.2). Figure 6.9 depicts the original distribution (A) and two altered distributions (B and C). In B, we truncated the original distribution by removing the variates of rank 1–5 and re-ranking the remaining variates; this treatment altered the slope considerably across the entire range, although the lower domain still appears approximately linear. Removal of every other variate from ranks 1 to 10, which is a form of censorship, produced the same effect on the slope in the lower domain. In C, we reduced the lake areas by an arbitrary proportion that was inversely related to their

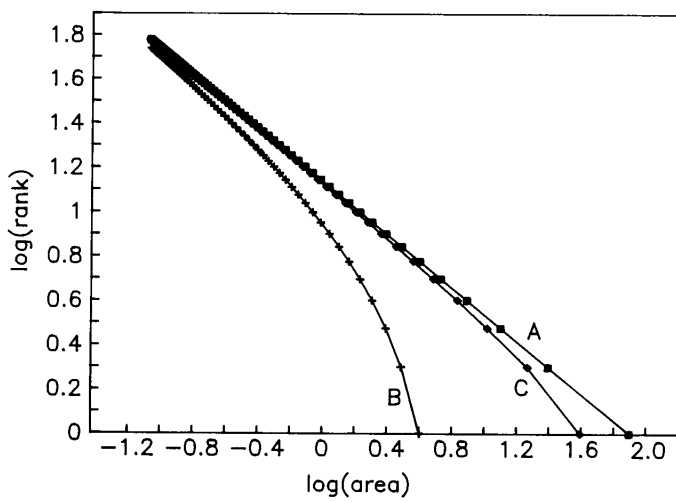

Figure 6.9 The effects of censorship and truncation demonstrated with an idealized Pareto distribution. See text for explanation

ranks; the largest lake was reduced by 50%, the second largest by 25%, the third by 17%, etc. This form of censorship had less effect on the slope. Clearly, censorship and truncation of the upper end can lead to biased estimates of the slopes of Pareto fits and, consequently, of the fractal dimension D.

The variation that we observed in slopes of the Pareto fits among lake classes may be caused simply by differences in the points of censorship and truncation, rather than by differences in the fractal dimension of the terrain. The approximate linearity of the Pareto fits suggests that fractal processes may determine the lake size distributions, but the limited area of floodplain on which the lakes can occur results in steeper slopes that cannot be related to the fractal dimension. The observed differences in Paretian slopes among lake classes seem consistent with this interpretation. On the Amazon floodplain, the slope is smallest for blocked-valley lakes (−0.6), intermediate for dish lakes (−1.0), and largest for channel lakes (−1.5). These differences may reflect different degrees of departure from the slope expected to result from the fractal dimension of the relief. Blocked-valley lakes might be expected to be the least constrained, since they occur outside of the floodplain boundaries. Dish lakes lie in back swamps, which occupy much of the floodplain, while channel lakes are restricted largely to the scroll-bar topography, which covers less area and tends to develop as discrete spatial units compared with the more continuous back swamps where dish lakes occur (Iriondo, 1982). On the Orinoco floodplain, dish and channel lakes appear less segregated (e.g. Figure 6.4); the similar slopes of Pareto fits for these lake classes may reflect their similar spatial constraints.

Comparisons with the FBM Model

Our original objective was to compare our empirical lake size distributions with those produced by the FBM simulation model. However, the parameters of the empirical distributions evidently are altered by censorship and truncation of the upper end. As Goodchild (1988) noted when he presented the Pareto fit for lake areas from the FBM model, the use of a limited area for simulation also results in censorship and truncation of the distribution. The parameters of the Pareto fits are therefore biased in both the simulation model and in the empirical observations, making meaningful comparisons difficult. The FBM model does appear to support Mandelbrot's assertions that fractal relief will have lakes whose areas follow the Pareto distribution. However, beyond that confirmation of fractal theory, its value as a 'null hypothesis' for the analysis of empirical data on lake-rich landscapes remains to be demonstrated.

CONCLUSIONS

Our measurements of the areas of lakes on the Amazon and Orinoco floodplains appear to follow the Pareto distribution, with censorship and truncation on both ends. This indicates that the lakes are statistically self-similar with respect to area over a wide range of observational scales, and suggests that the relief on the floodplain displays fractal characteristics. The observation that all of the lake classes show self-similarity is surprising, because certain lake classes are created by specific geomorphological processes that presumably operate at characteristic scales (e.g. Amazon

channel lakes in scroll-bar topography), while others are probably created by more scale-invariant processes (dish lakes).

Statistical self-similarity of lake areas implies that descriptive statistics for the lake populations, such as total abundance per unit land area and the medians and quartiles of lake areas, will vary with the scale of observation. Such statistics therefore cannot be compared between populations unless the same scales are used, or unless a fixed lower limit of lake size is adopted that lies above the resolution limit of the imagery. Statistical self-similarity with respect to area does not imply that the lakes are necessarily self-similar with respect to shape (i.e. that they are isometric rather than allometric (Church and Mark, 1980)).

Censorship and truncation of the upper end of a Pareto distribution are expected to occur for lake areas on floodplains because of the finite space in which the lakes occur. Spatial constraints similarly will affect any analysis of the size distributions of landforms, however, either because of a limited study area or because the landforms occur in a finite space. Consequently, the fractal dimension will be overestimated from size distributions of landforms observed in nature. Goodchild (1981, 1988) found that estimates of the fractal dimension obtained using real and simulated lake size distributions were inconsistent with those obtained by other independent methods, such as the area–perimeter relation. The present study reveals problems with the application of the size-distribution method to empirical data. These problems are likely to explain inconsistency of size-distribution analysis with other methods of estimating the fractal dimension.

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REFERENCES

Aitchison, J. and J. A. C. Brown (1963). *The Lognormal Distribution, with Special Reference to its Use in Economics*, Cambridge University Press, 194 pp.

Arnold, B. C. (1985). Pareto distribution. In *Encyclopedia of Statistical Sciences*, Vol. 6, Wiley, pp. 568–574.

Badger, W. W. (1980). An entropy–utility model for the size distribution of income. In B. J. West (ed.), *Mathematical Models as a Tool for the Social Sciences*, Gordon and Breach, pp. 87–120.

Blake, D. H. and C. D. Ollier (1971). Alluvial plains of the Fly River, Papua, *Zeitschrift für Geomorphologie, Supplementband*, **12**, 1–17.

Burrough, P. (1985). Fakes, fascimiles and facts: fractal models of geophysical phenomena. In S. Nash (ed.), *Science and Uncertainty*, Proceedings of a Conference Held Under the Auspices of IBM United Kingdom Ltd., London, March 1984, Science Reviews, pp. 151–169.

Church, M. and D. M. Mark (1980). On size and scale in geomorphology. *Progress in Physical Geography*, **4**, 342–390.

Departamento Nacional da Produção Mineral (1978). *Projeto RADAMBRASIL, Folha SA.20, Manaus: Geologia, Geomorfologia, Pedologia, Vegetação e Uso Potencial da Terra*, Departamento Nacional da Produção Mineral, Rio de Janeiro.

Goodchild, M. F. (1981). Randomness at Random Island: stochastic models of the physical landscape. Paper presented to the *Annual Meeting of the Canadian Association of Geographers*, Corner Brook, Newfoundland, August 1981.

Goodchild, M. F. (1988). Lakes on fractal surfaces: a null hypothesis for lake-rich landscapes. *Mathematical Geology*, **20**, 615–630.

Hamilton, S. K. and W. M. Lewis, Jr. (1990a). Physical characteristics of the fringing floodplain of the Orinoco River, Venezuela. *Interciencia*, **15**, 491–500.

Hamilton, S. K. and W. M. Lewis, Jr. (1990b). Basin morphology in relation to chemical and ecological characteristics of lakes on the Orinoco River floodplain, Venezuela. *Archiv für Hydrobiologie*, **119**, 393–425.

Hutchinson, G. E. (1957). *A Treatise on Limnology, 1: Geography, Physics and Chemistry*, Wiley, New York, and Chapman and Hall, London, 1015 pp.

Irion, G. (1984). Sedimentation and sediments of Amazonian rivers and evolution of the Amazonian landscape since Pliocene times. In H. Sioli (ed.), *The Amazon: Limnology and Landscape Ecology of a Mighty Tropical River and its Basin*, Monographiae Biologicae, Vol. 56, Dr. W. Junk, pp. 675–706.

Iriondo, M. H. (1982). Geomorfologia da planície Amazônica. In K. Suguio, M. R. Mousinho de Meis and M. G. Tessler (eds), *Atas do IV Simpósio do Quarternário no Brasil*, Sociedad Brasileira de Geologia, pp. 323–348.

Junk, W. J. (1983). Ecology of swamps on the middle Amazon. In A. J. P. Gore (ed.), *Ecosystems of the World 4B; Mires: Swamp, Bog, Fen and Moor; Regional Studies*, Elsevier, pp. 269–294.

Kaye, B. H. (1989). *A Randomwalk Through Fractal Dimensions*, VCH Verlagsgesellschaft, 421 pp.

Kent, C. and J. Wong (1982). An index of littoral zone complexity and its measurement. *Canadian Journal of Fisheries and Aquatic Sciences*, **39**, 847–853.

Klammer, G. (1984). The relief of the extra-Andean Amazon basin. In H. Sioli (ed.), *The Amazon: Limnology and Landscape Ecology of a Mighty Tropical River and its Basin*, Monographiae Biologicae, Vol. 56, Dr. W. Junk, pp. 47–83.

Korčak, J. (1940). Deux types fondamentaux de distribution statistique. *Bulletin de l'Institut International de Statistique*, **30**, 295–299.

Mandelbrot, B. (1977). *Fractals: Form, Chance and Dimension*, Freeman, 365 pp.

Mandelbrot, B. (1982). *The Fractal Geometry of Nature*, Freeman, 460 pp.

Meade, R. H., C. F. Nordin, Jr., D. P. Hernandez, A. Mejia and J. M. P. Godoy (1983). Sediment and water discharge in río Orinoco, Venezuela and Colombia. In *Proceedings of the Second International Symposium on River Sedimentation*, Water Resources and Electric Power Press, Beijing, pp. 1134–1144.

Meade, R. H., T. Dunne, J. E. Richey, U. de M. Santos and E. Salati (1985). Storage and remobilization of suspended sediment in the lower Amazon River of Brazil, *Science*, **228**, 488–490.

Melack, J. M. (1984). Amazon floodplain lakes: shape, fetch, and stratification. *Proceedings of the International Association of Theoretical and Applied Limnology*, **22**, 1278–1282.

Melack, J. M. and T. R. Fisher (1990). Comparative limnology of tropical floodplain lakes with an emphasis on the central Amazon. *Acta Limnologia Brasiliensia*, **3**, 1–48.

Mertes, L. A. K. (1985). *Floodplain Development and Sediment Transport in the Solimoes–Amazon River, Brazil*. Unpublished MS thesis, University of Washington, Seattle.

Milne, B. T. (1988). Measuring the fractal geometry of landscapes. *Applied Mathematics and Computation*, **27**, 67–79.

Richey, J. E., L. A. K. Mertes, T. Dunne, R. L. Victoria, B. R. Forsberg, A. C. N. S. Tancredi and E. Oliveira (1989). Sources and routing of the Amazon River flood wave. *Global Biogeochemical Cycles*, **3**, 191–204.

Salo, J. (1990). External processes influencing origin and maintenance of inland water–land ecotones. In R. Naiman and H. Décamps (eds), *The Ecology and Management of Aquatic–Terrestrial Ecotones*, Man and the Biosphere Series, Vol. 4, UNESCO and Parthenon, pp. 37–64.

Takayasu, H. (1990). *Fractals in the Physical Sciences*, Manchester University Press, 176 pp.

Tricart, J. (1977). Types de lits fluviaux en Amazonie brésilienne. *Annales de Géographie*, **86**, 1–54.

Welcomme, R. L. (1979). *Fisheries Ecology of Floodplain Rivers*, Longman, 317 pp.

West, B. J. and M. Shlesinger (1990). The noise in natural phenomena, *American Scientist*, **78**, 40–45.