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# Groundwater flux and open-channel estimation of stream metabolism: response to Hall and Tank

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### Abstract

Rates of metabolism for a stream can be estimated from open-channel measurements of dissolved oxygen at 1 or 2 stations, provided that reaeration flux and groundwater flux to the channel are quantified, and the underlying assumptions of the mass-balance method are satisfied. Hall and Tank (Limnol. Oceanogr. Methods 3:222-229, 2005) challenge our previous work on open-channel estimation of oxygen metabolism (McCutchan et al., Limnol. Oceanogr. 47:321-324, 2002) on grounds that our principal mass-balance equation is incorrect. However, Hall and Tank have misrepresented our equation by considering the oxygen mass-balance for an entire reach rather than for a parcel of water moving downstream and by presenting volume as a constant rather than a function of time. Because Hall and Tank incorrectly reformulated our mass-balance equation, their conclusions are not relevant to our work.

## Introduction

Hall and Tank (2005) present estimates of ecosystem metabolism for Giltner Spring Creek, a small stream that drains irrigated pasture in western Wyoming, USA. Rates of groundwater flux to Giltner Spring Creek are spatially variable but can exceed 3 m $d^{-1}$. Rates of this magnitude can significantly affect the oxygen mass-balance of a stream and should be considered in the estimation of ecosystem metabolism by open-channel methods (McCutchan et al. 2002; Hall and Tank 2005).

To estimate metabolism in Giltner Spring Creek, Hall and Tank (2005) employ a mass-balance equation that includes a term for groundwater flux to the stream. Their equation is an approximate solution to an equation given in McCutchan et al. (2002), which is based on the conceptual model originally presented by Odum (1956). Hall and Tank assert, however, that analyses presented in our paper of 2002 are incorrect because our principal mass-balance equation is incorrect. This assertion is false because Hall and Tank presented our mass-balance equation incorrectly and then analyzed the flaws in the incorrectly presented equation.

## Supporting information

Our previous analyses were based on the following mass-balance equation for a thin parcel of water moving downstream in a rectangular channel (Eq. 1, McCutchan et al. 2002):

$$\frac{dm}{dt} = C_g Q_g + (P + R)A + k(Sv[t] - m[t]) \qquad (1m)$$

where $dm/dt$ is the rate of change in mass ($m$) of oxygen over time ($t$), $C_g$ is the oxygen concentration of groundwater, and $Q_g$ is the rate of groundwater flux to the parcel (i.e., the rate of change in volume for the parcel). $P$ and $R$ are rates of photosynthesis and respiration, $A$ is the area of the channel covered by the parcel, $k$ is the reaeration rate coefficient for oxygen, and $S$ is the saturation concentration for oxygen. The volume of the parcel ($v[t]$) and the mass of oxygen in the parcel ($m[t]$) are functions of time. This equation describes an upstream-downstream (2-station) model developed from the approach that Odum (1956) used to estimate metabolism at Silver Springs, Florida, USA. Except for groundwater flux, water does not enter or leave the parcel as it flows downstream (i.e., it is assumed that the parcel does not gain or lose water in the upstream-downstream dimension and that direct precipitation and evaporation have a trivial influence on the volume of the parcel). It is assumed that the parcel represented in the equation corresponds to a thin slice of the stream and not to a study reach (Figure 1). The collection of data occurs at 2 stations (typically 0.1 to 1.0 km apart), and the rate of change applicable to the thin parcel is estimated from the measured change in concentration between the 2 stations. An important assumption implicit in the open-channel method is that the processes affecting the oxygen mass-balance for the parcel (i.e., metabolism, groundwater flux, and reaeration) are spatially homogeneous between the upstream and downstream stations. As the thin parcel moves downstream, volume of the parcel will change if there is net

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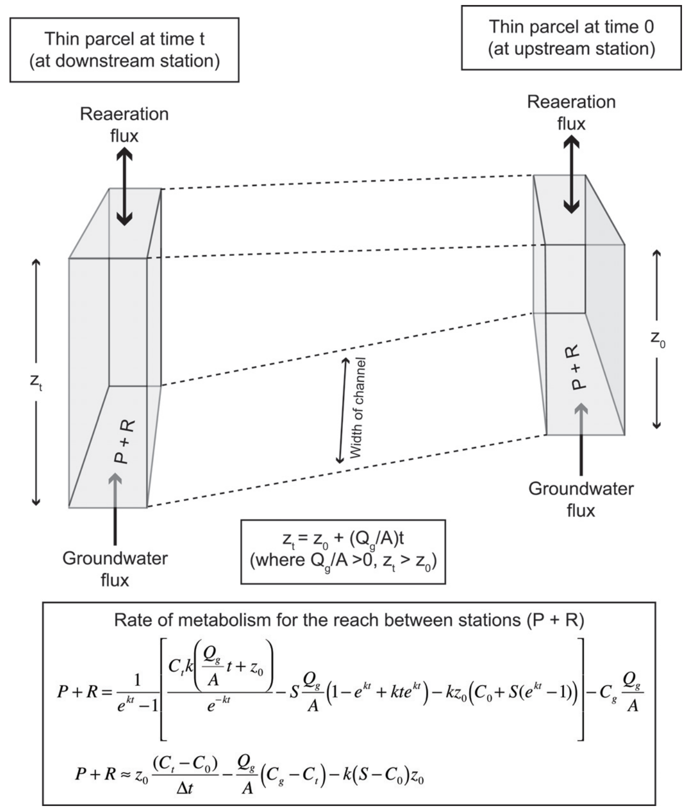


**Fig. 1.** Model of oxygen mass-balance for a parcel of water flowing downstream in a rectangular channel. Rates of metabolism for the reach between stations are calculated from the change in oxygen concentration for the parcel between upstream and downstream stations, after correction for reaeration flux and groundwater flux of oxygen (Eqs. 2m and 3m). In this example, the vertical velocity of groundwater flux ($Q_g/A$) is positive and $z_t > z_0$; if $Q_g/A$ were negative (as in a losing stream), depth for the parcel would decrease over the reach.

flux of groundwater across the sediment-water interface and the mass of oxygen in the parcel will change unless all fluxes of oxygen exactly cancel each other. Eq. 1m can be solved for $m[t]$, after which division by $v[t]$, followed by rearrangement, gives the rate of metabolism ($P + R$) as follows:

$$P + R = \frac{1}{e^{kt}-1}\left[\frac{C_t k\left(\frac{Q_g}{A}t + z_0\right)}{e^{-kt}} - S\frac{Q_g}{A}\left(1 - e^{kt} + kte^{kt}\right) - kz_0\left(C_0 + S(e^{kt}-1)\right)\right] - C_g\frac{Q_g}{A} \quad (2m)$$

where $C_t$ is the oxygen concentration at time $t$, $C_0$ is the oxygen concentration at time 0, and $z_0$ is the depth of the parcel at time 0. Alternatively, rates of metabolism can be estimated with a simplified equation (as in McCutchan et al. 2003 and Hall and Tank 2005) as follows:

$$P + R \approx z_0\frac{(C_t - C_0)}{\Delta t} - \frac{Q_g}{A}\left(C_g - C_t\right) - k\left(S - C_0\right)z_0 \quad (3m)$$

where $\Delta t$ is the travel time between stations. Over short intervals of time (i.e., travel time is small), estimates of metabolism calculated with Eq. 3m usually are similar (± 1%) to estimates calculated with Eq. 2m; if the reaeration rate is very high, however, estimates with Eq. 3m may differ substantially from estimates with Eq. 2m.

The conceptual model illustrated in Figure 1 reflects the LaGrangian nature of sampling with an upstream-downstream (2-station) application of the open-channel method. With a single-station (single-curve; Hall and Moll 1975) application of the open-channel method, it is tempting to envision the mass balance for a single, stationary reach in which the flow out of the reach downstream is balanced by flow into the reach from upstream and groundwater flux, but such a conceptual model is not correct because it cannot be assumed that the reach is well mixed in the upstream-downstream dimension. In fact, the upstream-downstream model illustrated in Figure 1 applies to single-station as well as 2-station applications (McCutchan et al. 2002). With the single-station approach, it is assumed that diel changes in temperature and oxygen concentration are identical at the station where measurements are taken and at a point upstream; the travel time between the point upstream and the station where measurements are taken is equal to the time interval between measurements (Hall and Moll 1975). With the upstream-downstream (2-station) approach, the distance between stations is fixed, but travel time between stations will vary if discharge is not constant (i.e., the mean velocity of flow changes). With the single-station (single-curve) approach, the time between measurements is constant, but the implicit reach length (i.e., the distance water travels over the time between measurements) will vary if discharge changes. Thus, upstream-downstream and single-station applications of the open-channel method both are based on an upstream-downstream model (as in Figure 1), but the single-station approach depends on the assumption of spatial homogeneity upstream and, with the single-station approach, the implicit length of the study reach varies as current velocity changes.

Hall and Tank (Eq. 1 in Hall and Tank 2005) give our Eq. 1m as follows:

$$\frac{dm}{dt} = C_g Q_g + MA + k(SAz - m) \quad (1h)$$

The terms for volume and mass in Eq. 1h (as defined in Table 1, Hall and Tank 2005: $A$ = reach area, $z$ = mean depth for a reach, and $m$ = mass of $O_2$ in a reach) differ from those of Eq. 1m (as defined above and in McCutchan et al. 2002). Hall and Tank define volume for a reach (the product of reach area and mean depth for the reach, $Az$) as a constant in Eq. 1h. In McCutchan et al. (2002), volume ($v[t]$) is defined for a thin parcel of water (not a reach) moving downstream and is presented as a function of time; if the vertical velocity of groundwater flux ($Q_g/A$ in m $d^{-1}$) is positive, depth and volume of the thin parcel will increase with time as the parcel moves downstream. In Eq. 1h, Hall and Tank consider the mass of oxygen ($m$) for an entire stream reach; McCutchan et al. (2002), in Eq. 1 (Eq. 1m), consider the mass of oxygen ($m[t]$) in a parcel of water moving downstream. Contrary to the statement by Hall and Tank that "the mass of oxygen will not really change from upstream to downstream," the mass of $O_2$ for the parcel ($m[t]$) will change as it moves downstream unless metabolism flux, reaeration flux, and groundwater flux of $O_2$ all sum to zero over a particular interval of time. The change in mass of oxygen in the parcel as it moves downstream is, in fact, the essence of the open-channel mass-balance method. Thus, Hall and Tank have misrepresented our equation by considering the oxygen mass-balance for an entire reach rather than for a parcel of water moving downstream and by presenting volume as a constant rather than a function of time. Hall and Tank are correct in stating that Eq. 1h from their paper is flawed, but it is not our equation, nor has anyone else proposed it as far as we know.

## Discussion

Upstream-downstream (2-station) and single-curve (single-station) applications of the open-channel method are both valid if their underlying assumptions are met. Two-station applications have the advantage that they do not require assumptions about conditions upstream of the stations where concentration is measured, but the single-curve approach is widely used because spatial homogeneity between distant stations often cannot be assumed (e.g., Rio La Mina; Ortiz-Zayas et al. 2005). With either approach, flux of groundwater to the channel can affect the mass balance for oxygen and, if flux of groundwater is ignored, substantial bias can result in estimates of metabolism. When accuracy is important with the open-channel method and especially when accurate estimates of ecosystem respiration are required, calculations of metabolism should be based on an equation that includes all of the terms that affect the mass balance for oxygen (e.g., Eq. 2m), as proposed by Odum (1956).

Our key mass-balance equation (Eq. 1m) is derived directly from the conceptual model presented by Odum (1956). An analytical solution to this equation (Eq. 2m) or a simplified approximation (Eq. 3m; as in McCutchan et al. 2003 and Hall and Tank 2005) can be used to estimate rates of ecosystem metabolism in running waters, but the criticisms that Hall and Tank (2005) have made of our previous work (McCutchan et al. 2002) are incorrect because Hall and Tank have presented our key mass-balance equation incorrectly.

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