Coriolis Force in the WRF model

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Equations

- $U(t+\Delta t) = U(t) + \Delta t * F_{U,Cor}$
- $V(t+\Delta t) = V(t) + \Delta t * F_{V,Cor}$
- $W(t+\Delta t) = W(t) + \Delta t * F_{W,Cor}$

- This is a forward in time method
- The WRF model describes both the horizontal and vertical Coriolis forces

Force Equations

- The forces $F_{U,Cor}$, $F_{V,Cor}$, $F_{W,Cor}$ are described by the following equations:
- $F_{U,Cor} = [(f_{i+1/2} + f_{i-1/2})/2]*[(V_{i+1/2,j+1/2} + V_{i+1/2,j-1/2} + V_{i-1/2,j+1/2} + V_{i-1/2,j-1/2})/4] – [(e_{i+1/2} + e_{i-1/2})/2]*[W_{i+1/2,k+1/2} + W_{i+1/2,k-1/2} + W_{i-1/2,k+1/2} + W_{i-1/2,k-1/2})/4]*[(\cos\alpha_{i+1/2} + \cos\alpha_{i-1/2})/2]$
- $F_{V,Cor} = -[(f_{j+1/2} + f_{j-1/2})/2]*[(U_{i+1/2,j+1/2} + U_{i+1/2,j-1/2} + U_{i-1/2,j+1/2} + U_{i-1/2,j-1/2})/4] – [(e_{j+1/2} + e_{j-1/2})/2]*[W_{j+1/2,k+1/2} + W_{j+1/2,k-1/2} + W_{j-1/2,k+1/2} + W_{j-1/2,k-1/2})/4]*[(\sin\alpha_{j+1/2} + \sin\alpha_{j-1/2})/2]$
- $F_{W,Cor} = e*\{[(U_{i+1/2,k+1/2} + U_{i+1/2,k-1/2} + U_{i-1/2,k+1/2} + U_{i-1/2,k-1/2})/4]*\cos\alpha - [(V_{j+1/2,k+1/2} + V_{j+1/2,k-1/2} + V_{j-1/2,k+1/2} + V_{j-1/2,k-1/2})/4]*\sin\alpha\}$

Force Equations

- These equations reduce to the following when we remove the grid staggering:

- $F_{U,Cor} = f*V - e*W*\cos\alpha$
- $F_{V,Cor} = -f*U - e*W*\sin\alpha$
- $F_{W,Cor} = e*U*\cos\alpha - V*\sin\alpha$

Force Equations

- This leaves us with the equations for the Coriolis Force as:

- $U(t+\Delta t) = U(t) + \Delta t * (f*V - e*W*\cos\alpha)$
- $V(t+\Delta t) = V(t) + \Delta t*(-f *U - e *W *\sin \alpha)$
- $W(t+\Delta t) = W(t) + \Delta t*(e*U *\cos \alpha - V *\sin \alpha)$

Force Equations

- where α is the local rotation angle between the y-axis and the meridians
- $e = 2\Omega\cos\varphi$
- $f = 2\Omega\sin\varphi$
- φ is the latitude
- Includes both horizontal and vertical effects

Stability Analysis

- Let $u = u_o \exp[i(kn\Delta x + \omega\tau\Delta t)]$
- $v = v_o \exp[i(kn\Delta x + \omega\tau\Delta t)]$
- $w = w_o \exp[i(kn\Delta x + \omega\tau\Delta t)]$
- and $\Psi = \exp(i\omega\Delta t)$. Plugging these values into the equations and putting it in matrix form we get:

Stability Analysis

$$\begin{vmatrix} \Psi\text{-}1 & -f\,\Delta t & e\cos\alpha\,\Delta t \\ f\,\Delta t & \Psi\text{-}1 & -e\sin\alpha\,\Delta t \\ -e\cos\alpha\,\Delta t & e\sin\alpha\,\Delta t & \Psi\text{-}1 \end{vmatrix} \begin{vmatrix} u_o \\ v_o \\ w_o \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

- We need to find the determinant of the matrix and set it equal to zero

Stability Analysis

- When we set the determinant equal to zero we get

$$(\Psi-1)[(\Psi-1)^2 + e^2\Delta t^2 + f^2\Delta t^2] = 0$$

- Now we need to solve for Ψ

Stability Analysis

- Solving for we get $\Psi = 1,\ 1 \pm i\ \Delta t\ \sqrt{(e^2+f^2)}$
- Equating real and imaginary parts we get:

  $\lambda\cos(\omega_r\Delta t) = 1$

  $\lambda\sin(\omega_r\Delta t) = \pm\Delta t\sqrt{(e^2+f^2)}$
- Squaring and summing we find that

  $\lambda^2 = 1 + \Delta t^2(e^2+f^2)$

Stability Analysis

- Solving for λ we find that $\lambda = \sqrt{[1+\Delta t^2(e^2+f^2)]} \geq 1$
- This shows that $\lambda = 1$ only when $\Delta t = 0$
- The scheme for the Coriolis force is unstable in WRF

References

Pielke, R.A., Sr., 2002: Mesoscale meteorological modeling. 2nd Edition, Academic Press, San Diego, CA, 676 pp.

Skamarock, W. C., J. B. Klemp, J. Dudhia, D. O. Gill, D. M. Barker, W. Wang, and J. G. Powers, 2005: A description of the Advanced Research WRF Version 2. NCAR Tech Notes-468+STR