



# The Weather Research & Forecasting Model WRF

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## The development of the Weather Research and Forecasting modeling system is a multiagency effort.

It is being developed as a collaborative effort among:

- the National Center for Atmospheric Research ([NCAR](#)) Mesoscale and Microscale Meteorology ([MMM](#)) division;
- the National Oceanic and Atmospheric Administration's ([NOAA](#)) National Centers for Environmental Prediction ([NCEP](#)) and Forecast System Laboratory ([FSL](#));
- the Department of Defense's Air Force Weather Agency ([AFWA](#)) and Naval Research Laboratory ([NRL](#));
- the Center for Analysis and Prediction of Storms ([CAPS](#)) at the OU;
- the Federal Aviation Administration ([FAA](#)).

The current WRF software framework supports two dynamical solvers: the Nonhydrostatic Mesoscale Model ([NMM](#)) developed by the NCEP and the Advanced Research WRF ([ARW](#)) developed and maintained by the Mesoscale and Microscale Meteorology Division of NCAR.

## Model Summary

- fully compressible Euler nonhydrostatic equations with hydrostatic option;
- scalar-conserving flux form for prognostic variables;
- complete Coriolis and curvature terms;
- nesting: one-way, two-way with multiple nests, moving nests;
- mass-based terrain following coordinate; vertical grid-spacing can vary with height;
- Mapping to Sphere: 3 map projections are supported for real-data simulations (Curvature terms included):

polar stereographic;  
Lambert-conformal;  
Mercator;

- Arakawa C-grid staggering;
- Runge-Kutta 2nd and 3rd order timestep options;
- 2nd to 6th order advection options (horizontal and vertical);
- positive-definite advection option for moisture, scalar and TKE;
- time-split small step for acoustic and gravity-wave modes;
- lateral boundary conditions:  
idealized cases: periodic, symmetric, and open radiative;  
real cases: specified with relaxation zone;
- full physics options for land-surface, PBL, radiation, microphysics and cumulus parameterization.

The Advanced WRF model current release is Version 3.0 (Apr. 2008).  
This version supports a variety of capabilities

These include:

- Real-data and idealized simulations;
- Various lateral boundary condition options for both real-data and idealized simulations;
- Full physics options;
- Non-hydrostatic and hydrostatic (runtime option);
- One-way, two-way nesting and moving nest;
- Applications ranging from meters to thousands of kilometers.

And (added with version 3.0)

- Global capability and rotated latitude-longitude grid option;
- Variable time step capability;
- Implicit upper boundary gravity-wave absorbing layer;
- New idealized test cases;
- Digital filter initialization;
- Options for surface field interpolations.

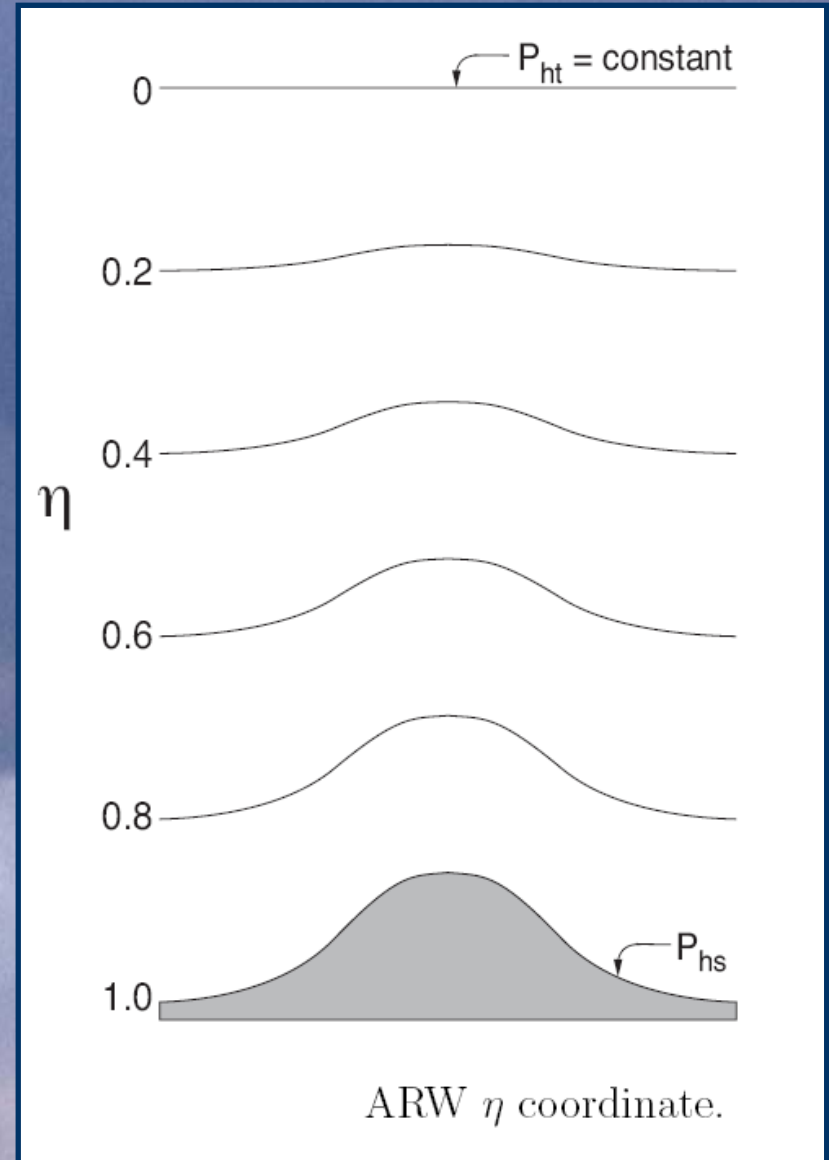
## (1/2) Vertical Coordinate and Variables

The ARW equations are formulated using a terrain-following hydrostatic-pressure vertical coordinate denoted by  $\eta$  and defined as:

$$\eta = (p_h - p_{ht}) / \mu \quad \text{where} \quad \mu = p_{hs} - p_{ht}.$$

$p_h$  is the hydrostatic component of the pressure, and  $p_{hs}$  and  $p_{ht}$  refer to values along the surface and top boundaries, respectively.

The coordinate definition, proposed by Laprise (1992), is the traditional  $\sigma$  coordinate used in many hydrostatic atmospheric models.

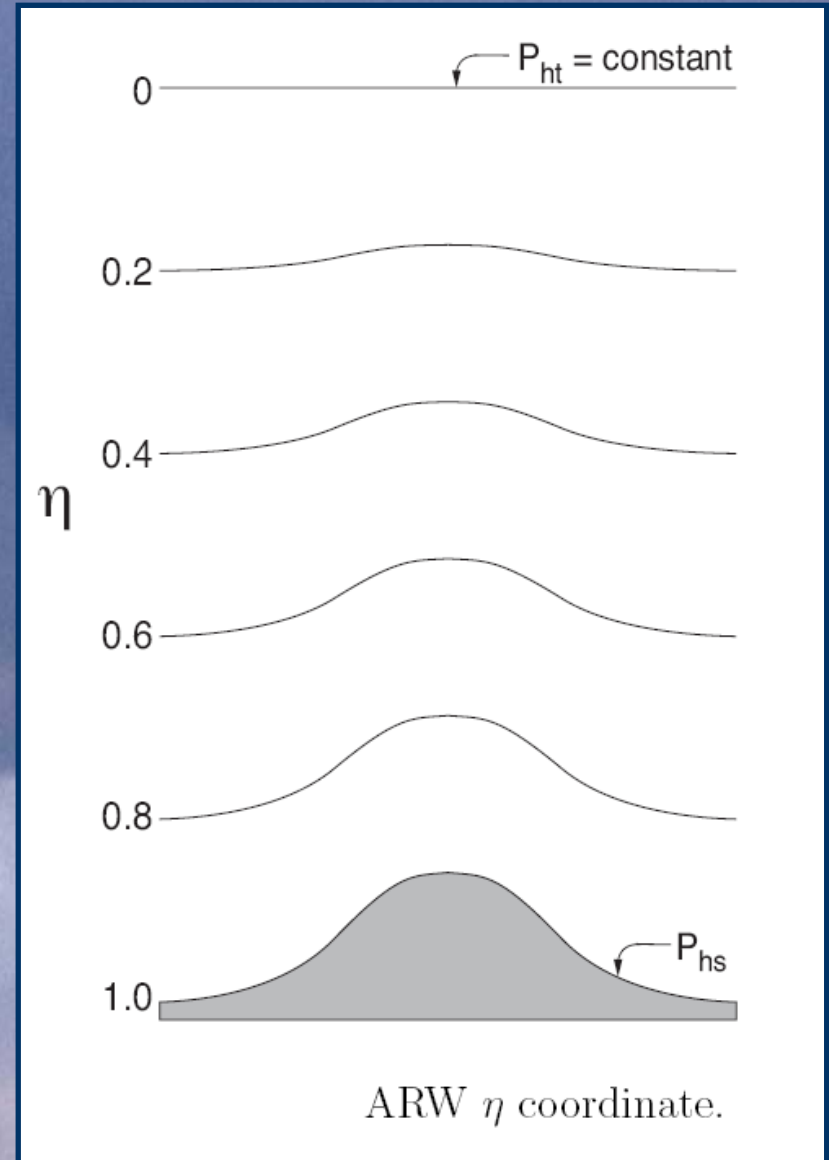


## (2/2) Vertical Coordinate and Variables

$\eta$  varies from a value of 1 at the surface to 0 at the upper boundary of the model domain. This vertical coordinate is also called a mass vertical coordinate.

Also appearing in the governing equations of the model are the non-conserved variables:

- $\phi = gz$  (the geopotential);
- $p$  (pressure);
- $\alpha = 1/\rho$  (the inverse density).



## Equations

For simplicity of interpretation we will view the flow in Cartesian coordinates and neglect the Coriolis effect. With these restrictions, the WRF model can be configured to solve the following equations:

$$p = \rho R_d T;$$

Equation of state

$$\frac{\partial \rho}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0;$$

Conservation of mass

$$\frac{\partial U}{\partial t} + c_p \Theta \frac{\partial \pi}{\partial x} = -\frac{\partial Uu}{\partial x} - \frac{\partial Vu}{\partial y} - \frac{\partial Wu}{\partial z} + F_x,$$

$$\frac{\partial V}{\partial t} + c_p \Theta \frac{\partial \pi}{\partial y} = -\frac{\partial Uv}{\partial x} - \frac{\partial Vv}{\partial y} - \frac{\partial Wv}{\partial z} + F_y,$$

Conservation of momentum

and

$$\frac{\partial W}{\partial t} + c_p \Theta \frac{\partial \pi}{\partial z} + g\rho = -\frac{\partial Uw}{\partial x} - \frac{\partial Vw}{\partial y} - \frac{\partial Ww}{\partial z} + F_z;$$

$$\frac{\partial \Theta}{\partial t} + \frac{\partial U\theta}{\partial x} + \frac{\partial V\theta}{\partial y} + \frac{\partial W\theta}{\partial z} = \rho Q.$$

Conservation of energy

and

$$U = \rho u, \quad V = \rho v, \quad W = \rho w, \quad \Theta = \rho \theta,$$

where  $(u, v, w)$  are the velocity components in the  $(x, y, z)$  directions,  $\theta$  is the potential temperature, and  $\rho$  is the air density. The other variables appearing above are the absolute temperature  $T$  and the Exner function  $\pi = (p/p_0)^{(R_d/c_p)}$ , where  $p$  is the pressure and  $p_0 = 1000$  hPa is a reference value. The specific heat at constant pressure for dry air is given by  $c_p = 1004.5 \text{ J K}^{-1} \text{ kg}^{-1}$ , and  $R_d = (2/7)c_p$  is the gas constant for dry air;  $F_x$ ,  $F_y$ , and  $F_z$  are friction terms.

## Runge-Kutta Time Integration Scheme

The RK3 scheme, described in Wicker and Skamarock (2002), integrates a set of ordinary differential equations using a predictor-corrector formulation. Defining the prognostic variables in the ARW solver as  $\Phi = (U, V, W, \Theta, \phi', \mu', Q_m)$  and the model equations as  $\Phi_t = R(\Phi)$ , the RK3 integration takes the form of 3 steps to advance a solution  $\Phi(t)$  to  $\Phi(t+\Delta t)$ :

$$\begin{aligned}\Phi^* &= \Phi^t + \frac{\Delta t}{3} R(\Phi^t) \\ \Phi^{**} &= \Phi^t + \frac{\Delta t}{2} R(\Phi^*) \\ \Phi^{t+\Delta t} &= \Phi^t + \Delta t R(\Phi^{**})\end{aligned}$$

where  $\Delta t$  is the time step for the low-frequency modes (the model time step). Superscripts denote time levels. This scheme is not a true Runge-Kutta scheme *per se* because, while it is third-order accurate for linear equations, it is only second-order accurate for nonlinear equations.

## RK3 Time Step Constraint

The RK3 time step is limited by the advective **Courant number** ( $u\Delta t/\Delta x$ ) and the user's choice of advection schemes — users can choose 2nd through 6th order discretizations for the advection terms. The time-step limitations for 1D advection in the RK3 scheme using these advection schemes is given in Wicker and Skamarock (2002), and is reproduced here.

Time Scheme	Spatial order			
	3rd	4th	5th	6th
Leapfrog	<i>Unstable</i>	0.72	<i>Unstable</i>	0.62
RK2	0.88	<i>Unstable</i>	0.30	<i>Unstable</i>
RK3	1.61	1.26	1.42	1.08

Maximum stable Courant numbers for one-dimensional linear advection.

As is indicated in the table, the maximum stable **Courant numbers** for advection in the RK3 scheme are almost a factor of two greater than those for the leapfrog time-integration scheme.

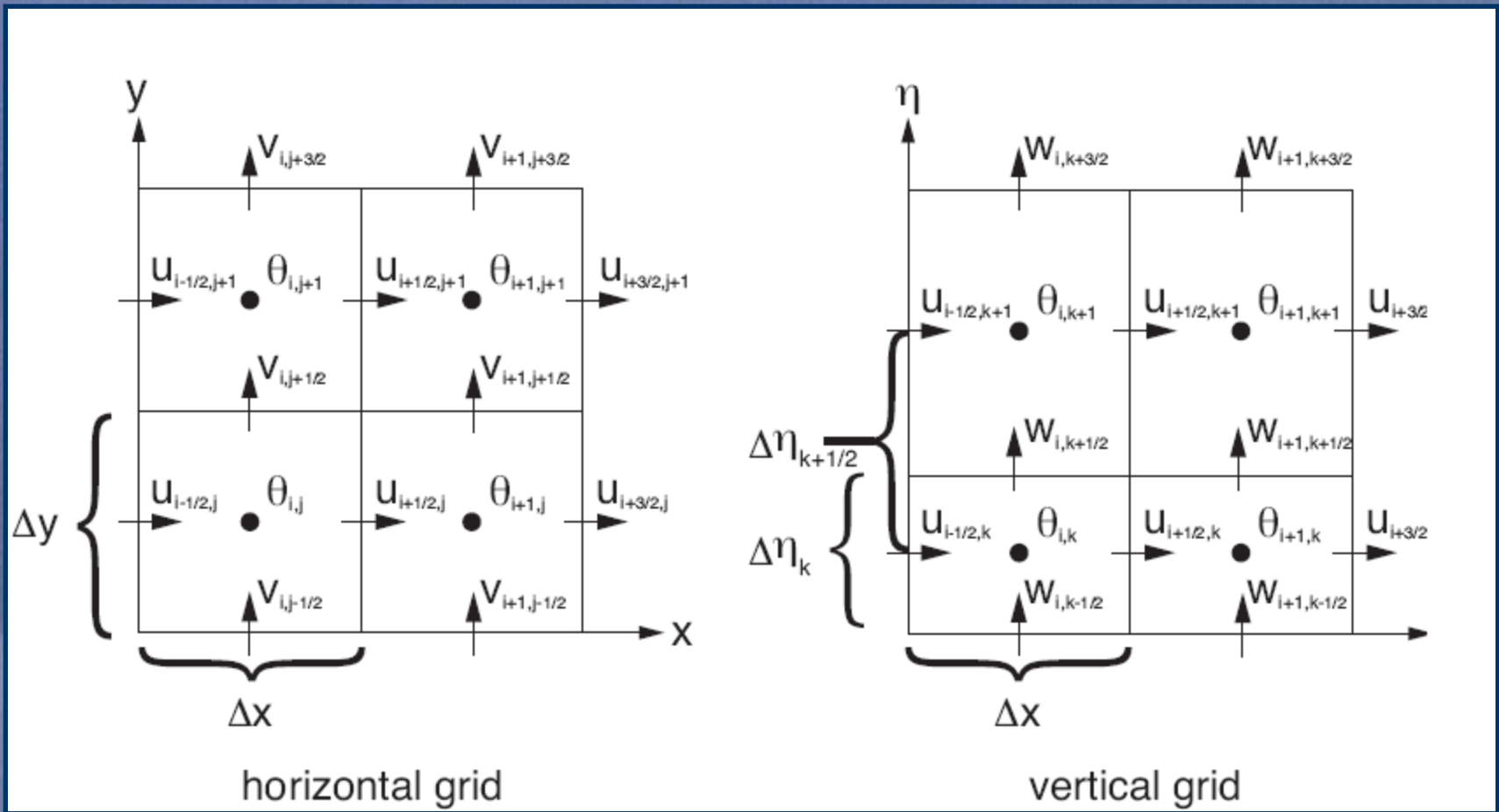
For advection in three spatial dimensions, the maximum stable Courant number is  $1/\sqrt{3}$  times the Courant numbers given in the Table. For stability, the time step used in the ARW should produce a maximum Courant number less than that given by theory. Thus, for 3D applications, the time step should satisfy the following equation:

$$\Delta t_{max} < \frac{Cr_{theory}}{\sqrt{3}} \cdot \frac{\Delta x}{u_{max}},$$

where  $Cr_{theory}$  is the Courant number taken from the RK3 entry in the Table and  $u_{max}$  is the maximum velocity expected in the simulation. For example in real-data applications, where jet stream winds may reach as high as 100 m/s, the maximum time step would be approximately 80 s on a  $\Delta x = 10$  km grid using 5th order advection. To provide a safety buffer, they usually choose a time step that is approximately 25% less than that given by the eq. This is typically a factor of two greater than that used in leapfrog-based models.

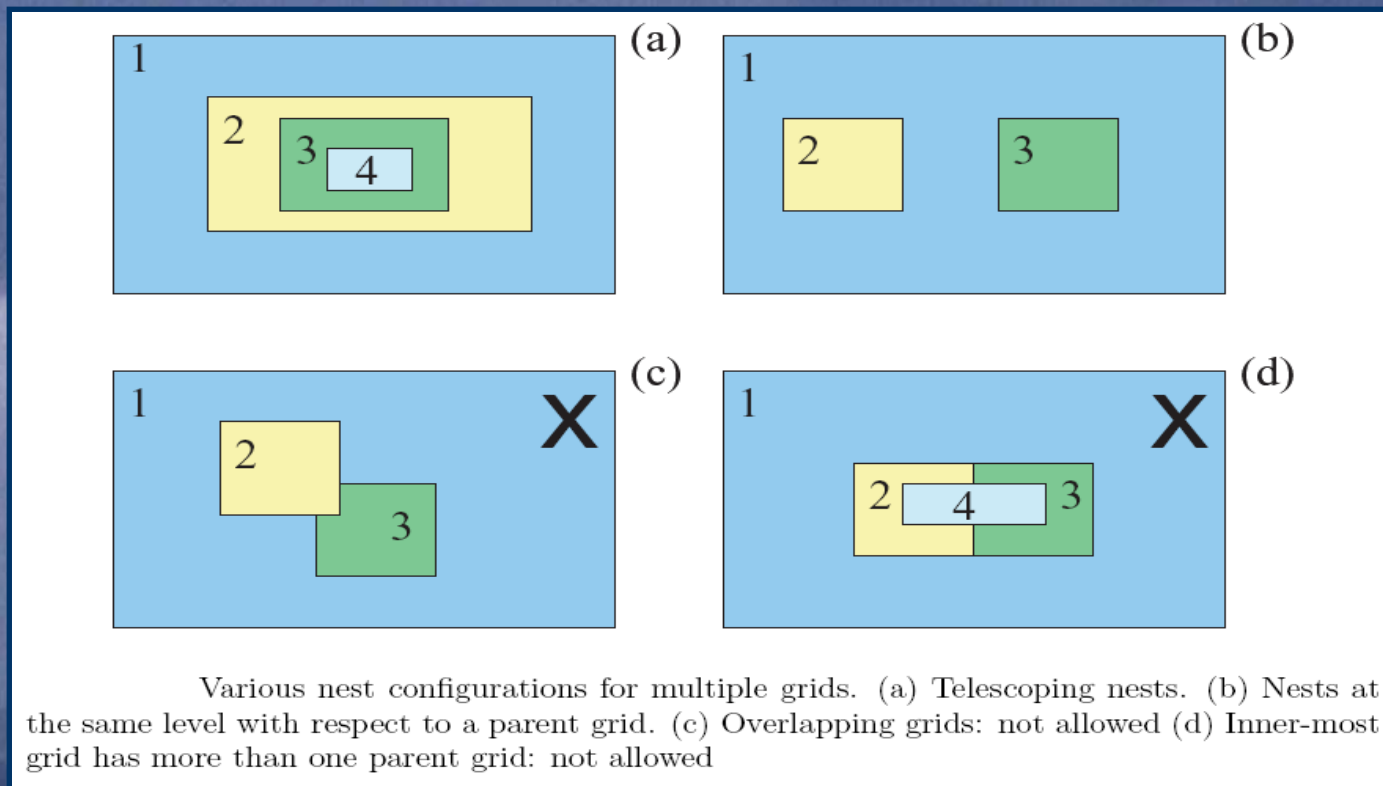
*For those users familiar with the MM5 model, the rule for choosing a time step is that the time step, in seconds, should be approximately 3 times the horizontal grid distance, in kilometers. For the ARW, the time step (in seconds) should be approximately 6 times the grid distance (in kilometers).*

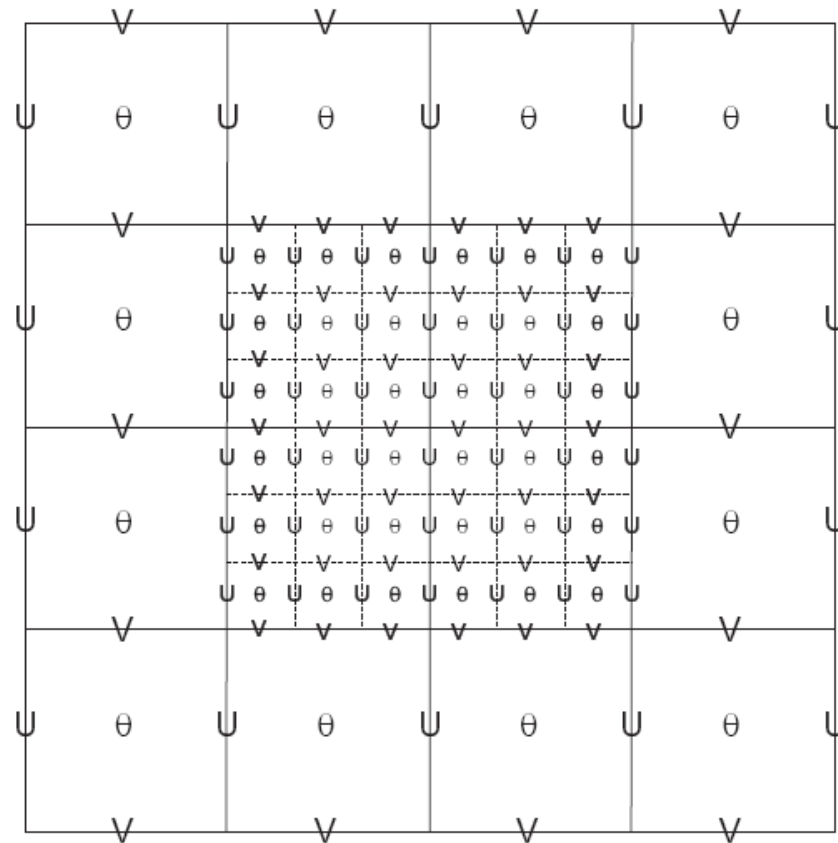
## The grid staggering is the Arakawa C-grid



The model uses a minimum grid spacing of 4 kilometers, with a total of 25 to 37 vertical divisions.

An ARW simulation involves one outer grid and may contain multiple inner nested grids. Each nested region is entirely contained within a single coarser grid (the parent grid). The finer, nested grids are referred to as child grids. Children are also parents when multiple levels of nesting are used. The fine grids may be telescoped to any depth and several fine grids may share the same parent at the same level of nesting. The fine grid may either be a static domain or it may be a moving nest with prescribed incremental shifts. Overlapping grids is not allowed. No grid can have more than a single parent.





Arakawa-C grid staggering for a portion of a parent domain and an imbedded nest domain with a 3:1 grid size ratio. The solid lines denote coarse grid cell boundaries, and the dashed lines are the boundaries for each fine grid cell. The horizontal components of velocity (“U” and “V”) are defined along the normal cell face, and the thermodynamic variables (“ $\theta$ ”) are defined at the center of the grid cell (each square). The bold typeface variables along the interface between the coarse and the fine grid define the locations where the specified lateral boundaries for the nest are in effect.

## Boundary conditions

- **Initial Conditions:**

Three dimensional for real-data, and one-, two- and three-dimensional using idealized data. A number of test cases are provided.

- **Top Boundary Conditions:**

Gravity wave absorbing (diffusion or Rayleigh damping).  $w = 0$   
Top boundary condition at constant pressure level.

- **Bottom Boundary Conditions:**

Physical or free-slip.

- **Lateral Boundary Conditions:**

Idealized cases: Periodic, open lateral radiative, and symmetric.  
Real cases: specified with relaxation zone.



## Fine Grid Initialization Options

The ARW supports several strategies to refine a coarse-grid simulation with the introduction of a nested grid. When using 1-way and 2-way nesting, several options for initializing the fine grid are provided.

- All of the fine grid variables can be interpolated from the coarse grid.
- All of the fine grid variables can be input from an external file which has high-resolution information for both the meteorological and the terrestrial fields.
- The fine grid can have some of the variables initialized with a high-resolution external data set, while other variables are interpolated from the coarse grid.

## Model Physics

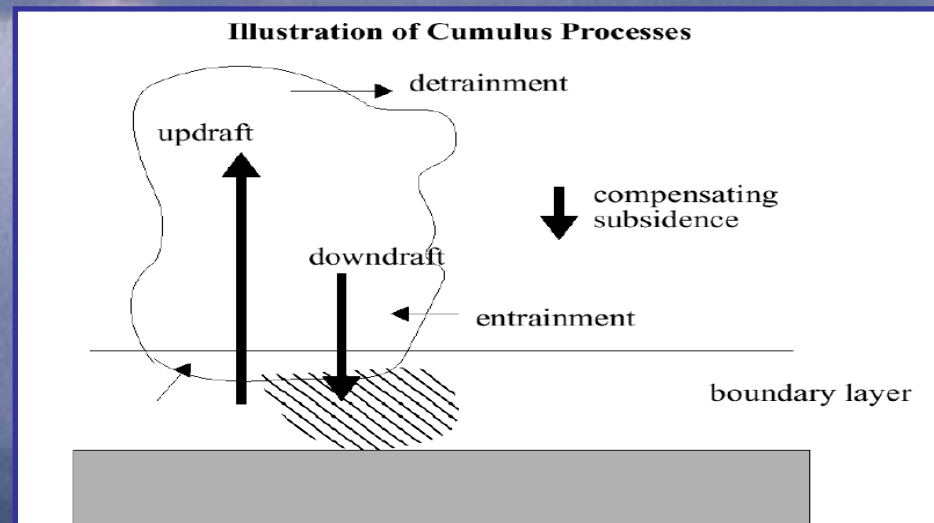
- Microphysics;
- Cumulus parameterizations;
- Surface physics;
- Planetary boundary layer physics;
- Atmospheric radiation physics.

## Microphysics (mp\_physics)

- Kessler scheme: A warm-rain (i.e. no ice) scheme used commonly in idealized cloud modeling studies (mp\_physics = 1).
- Lin et al. scheme: A sophisticated scheme that has ice, snow and graupel processes, suitable for real-data high-resolution simulations (2).
- WRF Single-Moment 3-class scheme: A simple efficient scheme with ice and snow processes suitable for mesoscale grid sizes (3).
- WRF Single-Moment 5-class scheme: A slightly more sophisticated version of the previous that allows for mixed-phase processes and super-cooled water (4).
- Eta microphysics: The operational microphysics in NCEP models. A simple efficient scheme with diagnostic mixed-phase processes (5).
- WRF Single-Moment 6-class scheme: A scheme with ice, snow and graupel processes suitable for high-resolution simulations (6).
- Thompson et al. scheme: A new scheme with ice, snow and graupel processes suitable for high-resolution simulations (8; replacing the version in 2.1).
- NCEP 3-class: An older version of (3) (98).
- NCEP 5-class: An older version of (4) (99).

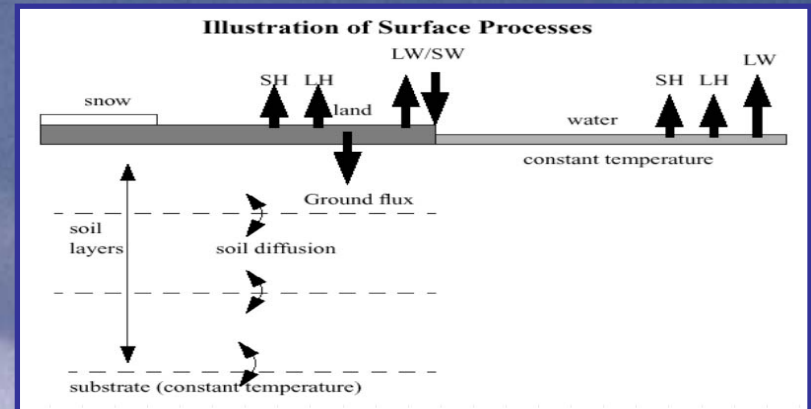
## Cumulus Parameterization (cu\_physics)

- Kain-Fritsch scheme: Deep and shallow convection sub-grid scheme using a mass flux approach with downdrafts and CAPE removal time scale (cu\_physics = 1).
- Betts-Miller-Janjic scheme: Operational Eta scheme. Column moist adjustment scheme relaxing towards a well-mixed profile (2).
- Grell-Devenyi ensemble scheme: Multi-closure, multi-parameter, ensemble method with typically 144 sub-grid members (3).
- Old Kain-Fritsch scheme: Deep convection scheme using a mass flux approach with downdrafts and CAPE removal time scale (99).



## Surface Layer (sf\_sfclay\_physics)

- **MM5 similarity:** Based on Monin-Obukhov with Carlson-Boland viscous sub-layer and standard similarity functions from look-up tables (sf\_sfclay\_physics = 1).
- **Eta similarity:** Used in Eta model. Based on Monin-Obukhov with Zilitinkevich thermal roughness length and standard similarity functions from look-up tables (2).

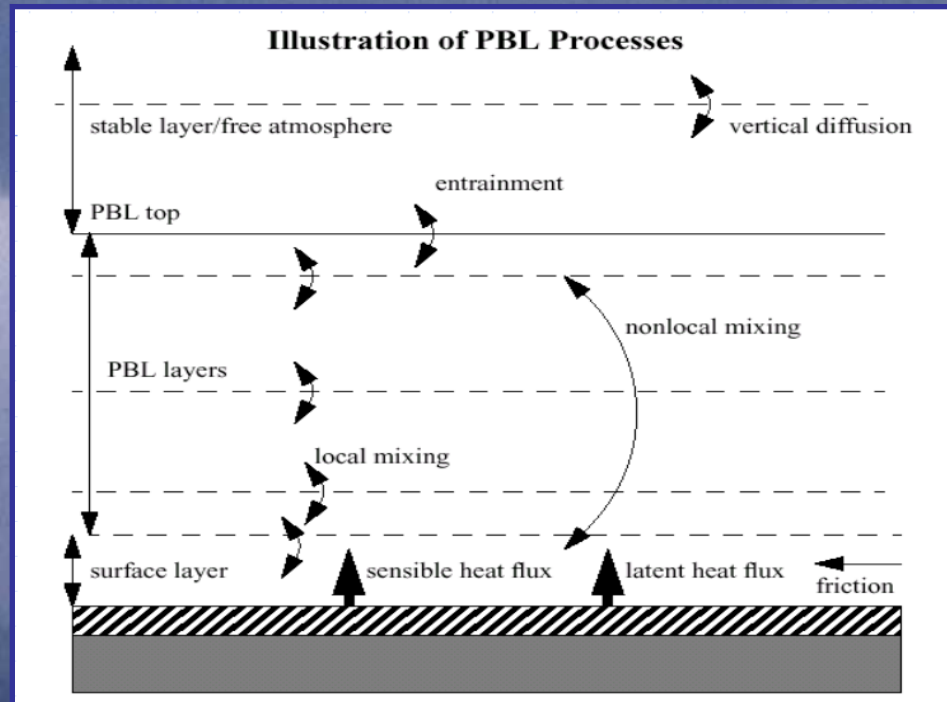


## Land Surface (sf\_surface\_physics)

- **5-layer thermal diffusion:** Soil temperature only scheme, using five layers (sf\_surface\_physics = 1).
- **Noah Land Surface Model:** Unified NCEP/NCAR/AFWA scheme with soil temperature and moisture in four layers, fractional snow cover and frozen soil physics (2).
- **RUC Land Surface Model:** RUC operational scheme with soil temperature and moisture in six layers, multi-layer snow and frozen soil physics (3).

## Planetary Boundary layer (bl\_pbl\_physics)

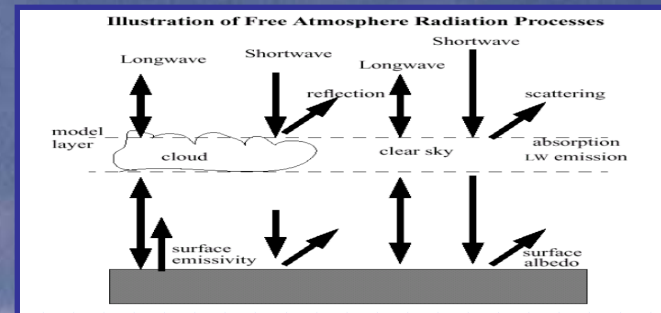
- Yonsei University scheme: Non-local-K scheme with explicit entrainment layer and parabolic K profile in unstable mixed layer (`bl_pbl_physics = 1`).
- Mellor-Yamada-Janjic scheme: Eta operational scheme. One-dimensional prognostic turbulent kinetic energy scheme with local vertical mixing (2).
- MRF scheme: Older version of (1) with implicit treatment of entrainment layer as part of non-local-K mixed layer (99).



## Longwave Radiation (ra\_lw\_physics)

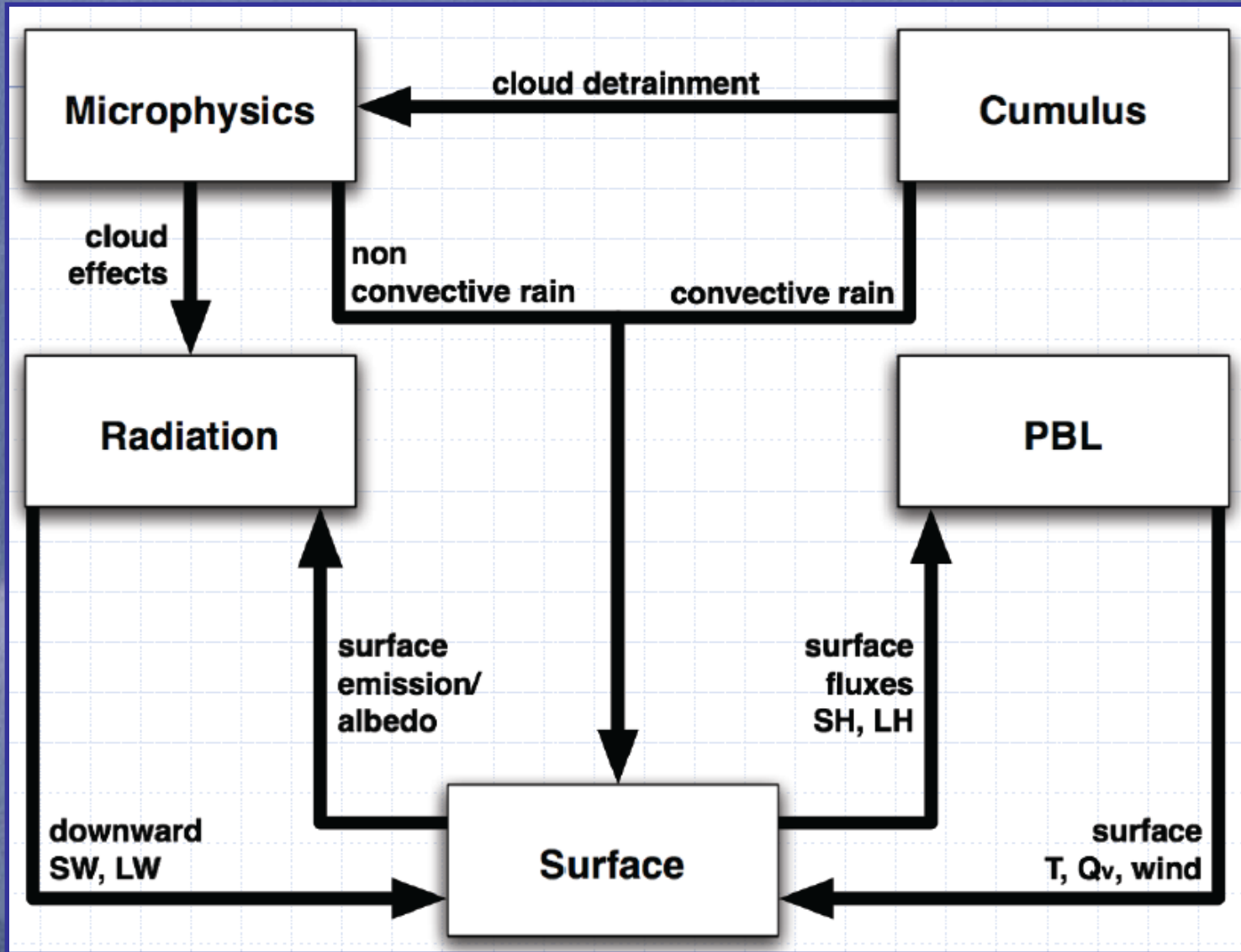
- RRTM scheme: Rapid Radiative Transfer Model. An accurate scheme using look-up tables for efficiency. Accounts for multiple bands, trace gases, and microphysics species (ra\_lw\_physics = 1).
- GFDL scheme: Eta operational radiation scheme. An older multi-band scheme with carbon dioxide, ozone and microphysics effects (2).
- CAM scheme: from the CAM 3 climate model used in CCSM. Allows for aerosols and trace gases (3).

## Shortwave Radiation (ra\_sw\_physics)

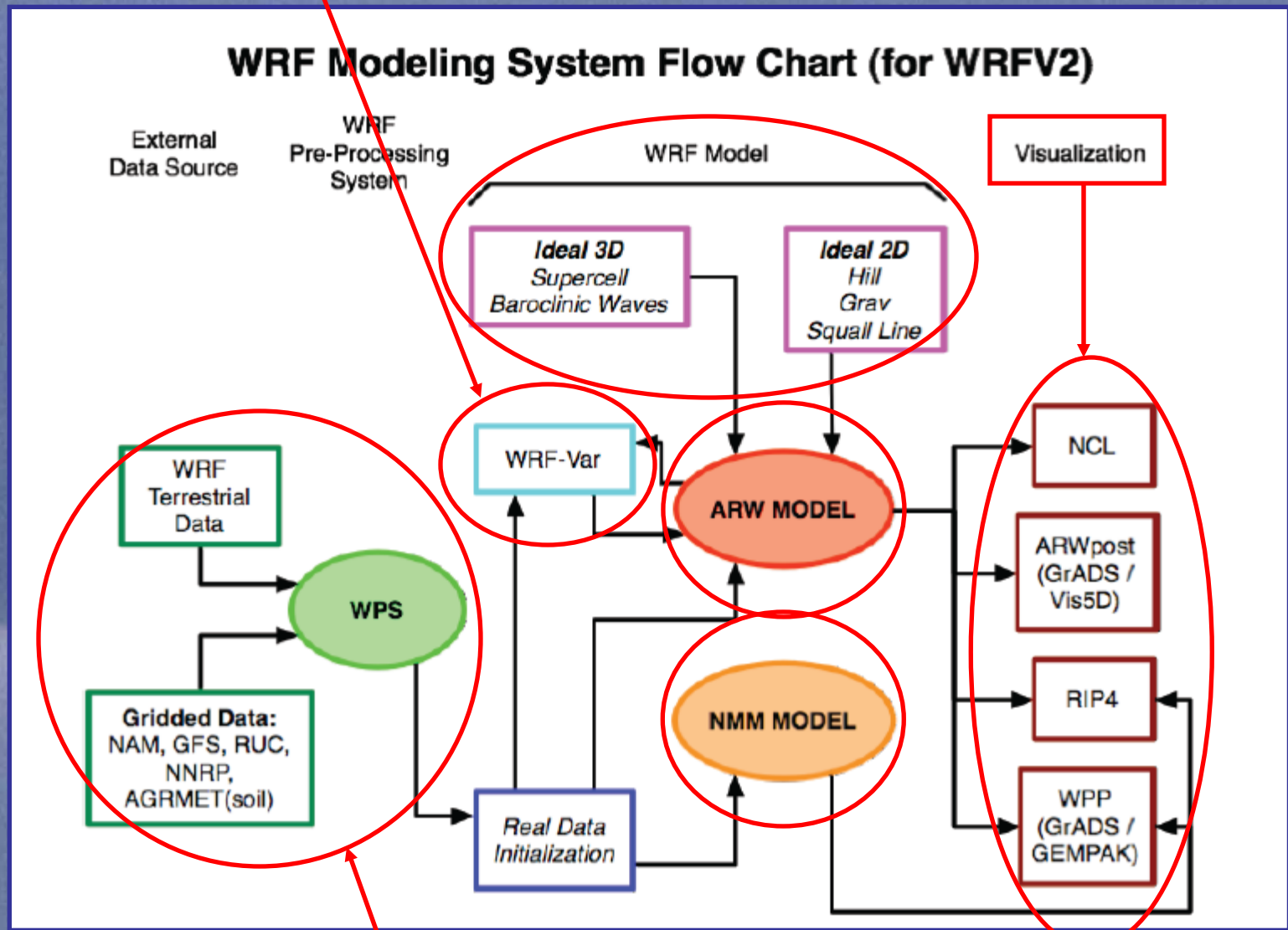


- Dudhia scheme: Simple downward integration allowing efficiently for clouds and clear-sky absorption and scattering (ra\_sw\_physics = 1).
- Goddard shortwave: Two-stream multi-band scheme with ozone from climatology and cloud effects (2).
- GFDL shortwave: Eta operational scheme. Two-stream multi-band scheme with ozone from climatology and cloud effects (99).
- CAM scheme: from the CAM 3 climate model used in CCSM. Allows for aerosols and trace gases (3).

## Direct Interaction of Parameterizations



WRF-ARW only variational data assimilation



The purpose of the WRF Processing System (WPS) is to prepare input to WRF for real-data simulations. To run the model, it needs to be initialized with the current atmospheric state. This is done with the analysis generated by a coarser, larger scale NWP model such as the Nam or GFS

## References

The following papers describe the equations and numerical schemes used in the WRF ARW core.

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