

*Project 2 - ATMO 595E*

Louis Surface Flux Scheme &  
Look-Up Table Approach for Parameterization

(Revised)

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# *Outline*

- Louis Surface Flux Scheme
- Look-Up Table (LUT) for Parameterization
- Understandings & Suggestions about LUT Approach

## Surface fluxes in model equations

$$\frac{\partial u}{\partial t} = \left\{ \frac{\partial}{\partial x} (-\overline{u'u'}) + \frac{\partial}{\partial y} (-\overline{u'v'}) + \frac{\partial}{\partial z} (-\overline{u'w'}) \right\} + \{advection\} + \{PGF + Coriolis\}$$

$$\frac{\partial \theta}{\partial t} = \left\{ \frac{\partial}{\partial x} (-\overline{\theta'u'}) + \frac{\partial}{\partial y} (-\overline{\theta'v'}) + \frac{\partial}{\partial z} (-\overline{\theta'w'}) \right\} + \{advection\} + \{Diabatic\}$$

$$\frac{\partial r}{\partial t} = \left\{ \frac{\partial}{\partial x} (-\overline{r'u'}) + \frac{\partial}{\partial y} (-\overline{r'v'}) + \frac{\partial}{\partial z} (-\overline{r'w'}) \right\} + \{advection\}$$

## Surface momentum flux - Louis scheme

$$(-\overline{u'w'})_0 \equiv u_*^2 = a^2 \overline{u}^2 F_m \left( \frac{z}{z_0}, R_{iB} \right)$$

$$R_{iB} = \frac{gz}{u^2} \frac{\Delta\theta}{\theta} = \frac{gz}{u^2} \frac{(\theta - \theta_0)}{\theta}$$

Bulk Richardson number (stability)

## Surface heat flux - Louis scheme

$$(-\overline{\theta'w'})_0 \equiv \theta_* u_* = \frac{a^2}{R} u (\theta - \theta_0) F_h \left( \frac{z}{z_0}, R_{iB} \right)$$

$$R_{iB} = \frac{gz}{u^2} \frac{\Delta\theta}{\theta} = \frac{gz}{u^2} \frac{(\theta - \theta_0)}{\theta}$$

$\theta_0$  surface temperature

## Surface vapor flux – Louis scheme

$$(-\overline{r'w'})_0 \equiv r_* u_* = \frac{a^2}{R} u (r - r_0) F_h \left( \frac{z}{z_0}, R_{iB} \right)$$

$$R_{iB} = \frac{gz}{u^2} \frac{\Delta\theta}{\theta} = \frac{gz}{u^2} \frac{(\theta - \theta_0)}{\theta}$$

$r_0$  surface moisture

## Expressions of $F$

stable

$$F = 1 - \frac{bR iB}{1 + c |R iB|^{1/2}}$$

unstable

$$F = \frac{1}{(1 + b' R iB)^2}$$

# Constants

stable  $F = \frac{1}{(1 + b'R_{iB})^2}$   $b' = 4.7$

$$R_{iB} = \frac{gz}{u^2} \frac{(\theta - \theta_0)}{\theta}$$

unstable  $F = 1 - \frac{bR_{iB}}{1 + c|R_{iB}|^{1/2}}$   $b = 9.4$

$$a^2 = \frac{k^2}{(\ln \frac{z}{z_0})^2}$$

$$c = C^* a^2 b \left(\frac{z}{z_0}\right)^{1/2}$$

$$C^* = 7.4 \quad F_m$$

$$C^* = 5.3 \quad F_h$$

## Momentum flux - stable

$$(-\overline{u'w'})_0 = \frac{k^2 u^2}{\left(\ln \frac{z}{z_0}\right)^2 \left(1 + 4.7 \frac{gz}{u^2} \frac{\theta - \theta_0}{\theta}\right)^2} = \left\{ \begin{array}{l} u, \theta \\ z \\ \theta_0, z_0 \\ 4.7 \\ g, k \end{array} \right\}$$

## Momentum flux - unstable

$$(-\overline{u'w'})_0 = \frac{k^2 u^2}{\left(\ln \frac{z}{z_0}\right)^2} \left\{ 1 - \frac{9.4 \frac{gz}{u^2} \frac{\theta - \theta_0}{\theta}}{1 + 7.4 \frac{k^2}{\left(\ln \frac{z}{z_0}\right)^2} 9.4 \left(\frac{z}{z_0}\right)^{1/2} \left| \frac{gz}{u^2} \frac{\theta - \theta_0}{\theta} \right|^{1/2}} \right\} = \left\{ \begin{array}{l} u, \theta \\ z \\ \theta_0, z_0 \\ 7.4, 9.4 \\ g, k \end{array} \right\}$$

## Heat flux - stable

$$(-\overline{\theta'w'})_0 = \frac{k^2 u (\theta - \theta_0)}{\left(\ln \frac{z}{z_0}\right)^2 R \left[1 + 4.7 \frac{gz}{u^2} \frac{\theta - \theta_0}{\theta}\right]^2} = \left\{ \begin{array}{l} u, \theta \\ z \\ \theta_0, z_0 \\ 4.7 \\ R \\ g, k \end{array} \right\}$$

# Heat flux - unstable

$$(-\overline{u'w'})_0 = \frac{k^2 u (\theta - \theta_0)}{(\ln \frac{z}{z_0})^2 R} \left\{ 1 - \frac{9.4 \frac{gz}{u^2} \frac{\theta - \theta_0}{\theta}}{1 + 5.3 \frac{k^2}{(\ln \frac{z}{z_0})^2} 9.4 (\frac{z}{z_0})^{1/2} \left| \frac{gz}{u^2} \frac{\theta - \theta_0}{\theta} \right|^{1/2}} \right\} = \left\{ \begin{array}{l} u, \theta \\ z \\ \theta_0, z_0 \\ 5.3, 9.4 \\ R \\ g, k \end{array} \right\}$$

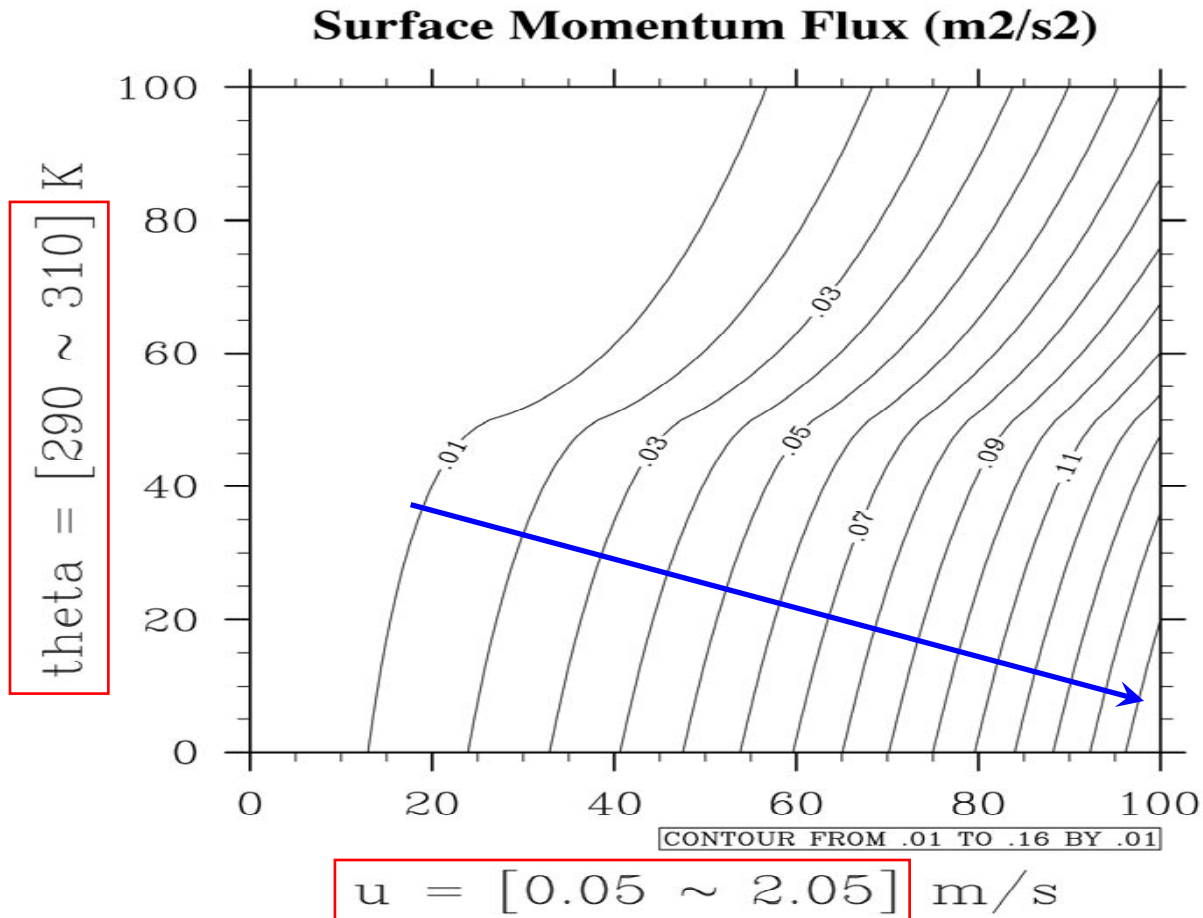
## Vapor flux - stable

$$(-\overline{r'w'})_0 = \frac{k^2 u (r - r_0)}{\left(\ln \frac{z}{z_0}\right)^2 R \left(1 + 4.7 \frac{g z (\theta - \theta_0)}{u^2 \theta}\right)^2} = \left\{ \begin{array}{l} u, \theta, r \\ z \\ \theta_0, r_0, z_0 \\ 4.7 \\ R \\ g, k \end{array} \right\}$$

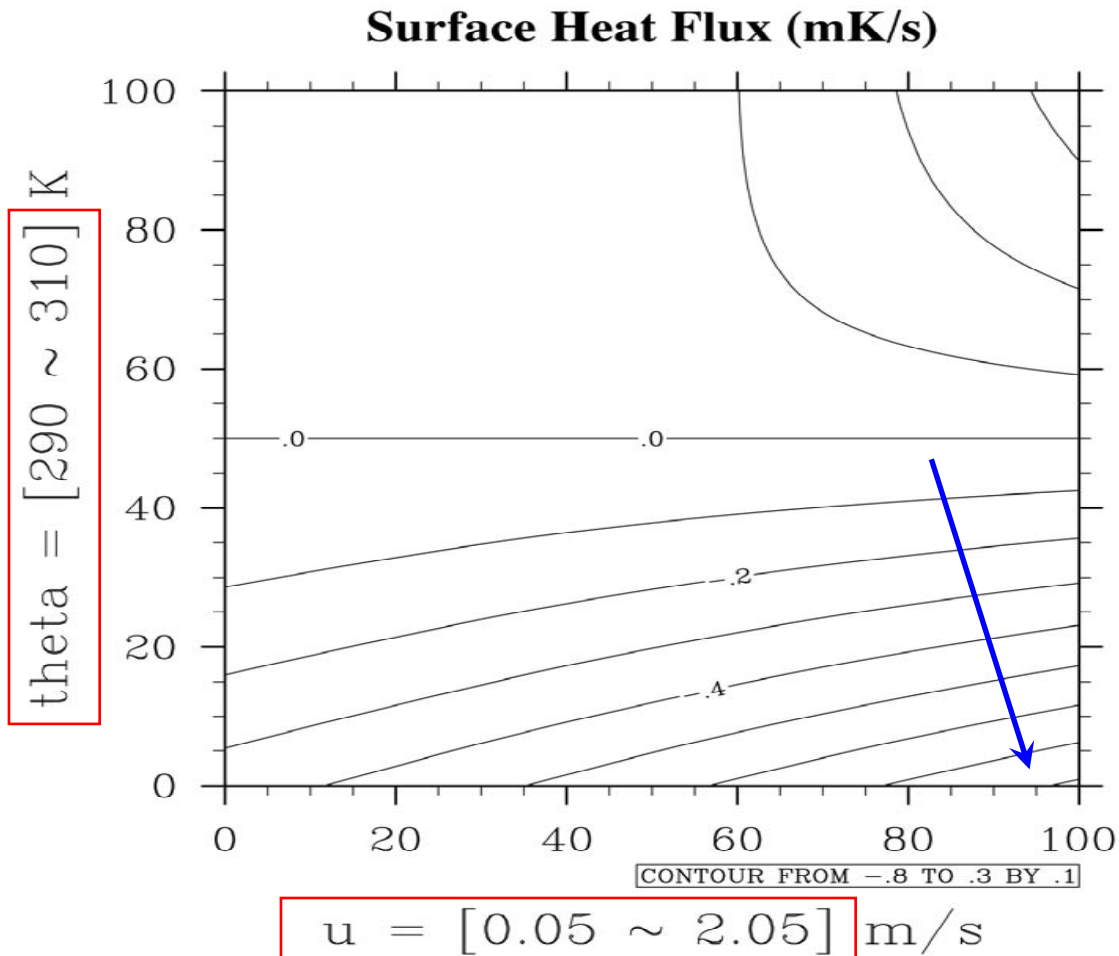
# Vapor flux - unstable

$$(-\overline{u'w'})_0 = \frac{k^2 u (r - r_0)}{(\ln \frac{z}{z_0})^2 R} \left\{ 1 - \frac{9.4 \frac{gz}{u^2} \frac{\theta - \theta_0}{\theta}}{1 + 5.3 \frac{k^2}{(\ln \frac{z}{z_0})^2} 9.4 (\frac{z}{z_0})^{1/2} \left| \frac{gz}{u^2} \frac{\theta - \theta_0}{\theta} \right|^{1/2}} \right\} = \left\{ \begin{array}{l} u, \theta, r \\ z \\ \theta_0, r_0, z_0 \\ 5.3, 9.4 \\ R \\ g, k \end{array} \right\}$$

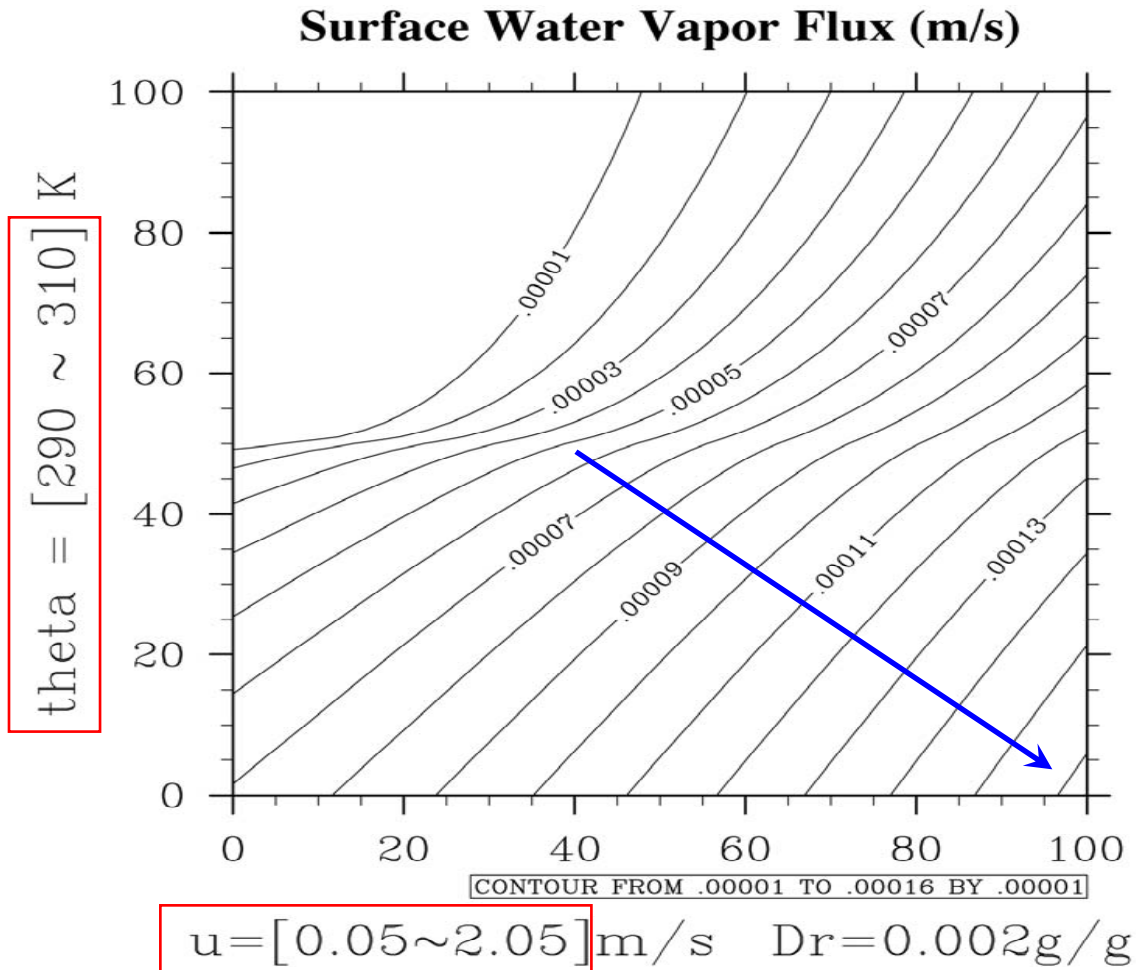
# Look-Up Table (plot) from source code



# Look-Up Table (plot) from source code



# Look-Up Table (plot) from source code



## Question 1

- Can we use a single LUT for the model with all the variables included?

## A general LUT (if storage allows)

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z} + -\theta \frac{\partial \pi'}{\partial x} + fv + \left\{ \frac{\partial}{\partial x} (-\overline{u'u'}) + \frac{\partial}{\partial y} (-\overline{u'v'}) + \frac{\partial}{\partial z} (-\overline{u'w'}) \right\}$$

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z} + -\theta \frac{\partial \pi'}{\partial x} + fv + P(u, \theta; \theta_0, z_0; \Delta x, \Delta z, \dots)$$

$$u^{\tau+1} = u^{\tau} + \Delta t \left\{ -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z} + -\theta \frac{\partial \pi'}{\partial x} + fv + P(u, \theta; \theta_0, z_0; \Delta x, \Delta z, \dots) \right\}^{\tau}$$

$$u^{\tau+1} = Q^{\tau} (u, v, w, \theta, \pi'; \theta_0, z_0, \Delta x, \Delta z; \Delta t, \dots)$$

- No calculation is needed if using this general LUT to do model integrations step by step.
- Too many variables involved in the LUT. It is not feasible for current computer storage. So multi-LUTs are needed.

## Question 2

- Can we use a lot of LUTs with just a few variables in each LUT?

No. If so, we still need to do too many calculations, and the LUT method becomes less meaningful.

## Question 3

- How to balance the number of the LUTs and the complexity (size or elements) of each LUT?

## Balance between the size and number of LUT

$$P(x_1, x_2, x_3, x_4, x_5, x_6) = A(x_1, x_2) + B(x_3, x_4)C(x_5, x_6)$$

$$\left. \begin{array}{l} \text{LUT of A} \quad n \times n \\ \text{LUT of B} \quad n \times n \\ \text{LUT of C} \quad n \times n \end{array} \right\} 3n^2 = 3 \times 10^6 \quad (n = 1000)$$

$$\text{LUT of P} \quad n \times n \times n \times n \times n \times n = n^6 = 10^{18}$$

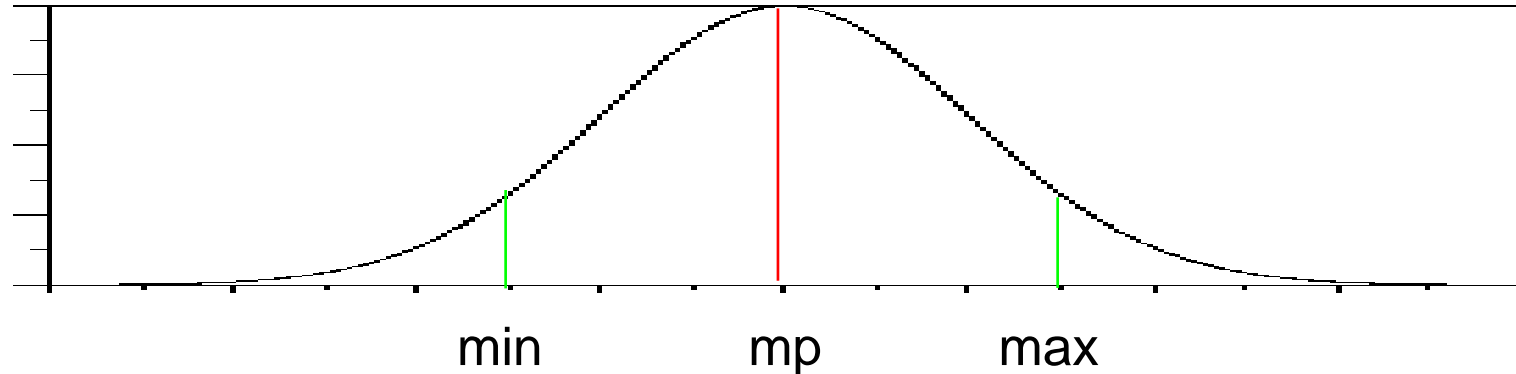
The number of LUTs and the elements of each LUT should be balanced.

- If the terms or factors in a parameterization have less variables in common, create LUTs separately for them.
- A LUT should use less variables, but do more calculations.

## Question 4

- In a LUT, how to determine the domain (min and max) of each variable?

## *Set domain for each variable in LUT*



- Analyze the distribution of each variable. Determine its most popular value.
- Set the domain (min and max) of a variable with limit of computer storage considered.
- For values out of the domain, calculate through parameterization schemes.

## Question 5

- In a LUT, the spacings (or the mean spacings) of different variables can be chosen separately, or should be related?

## Set spacing for each variable in LUT

Suppose a parameterization scheme  $P = P(u, v, \dots)$

$$\Delta P(u, v) = \frac{\partial P}{\partial u} \Delta u + \frac{\partial P}{\partial v} \Delta v + \dots$$

Variable  $u$  and  $v$  may have different effects on  $P$  ( $\frac{\partial P}{\partial u}$  and  $\frac{\partial P}{\partial v}$ ). Let  $\Delta u$  and  $\Delta v$  be the spacings of  $u$  and  $v$ . Consider the changes of  $u$  and  $v$  can cause same contributions to the change of  $P$ , that is

$$\frac{\partial P}{\partial v} \Delta v = \frac{\partial P}{\partial u} \Delta u$$

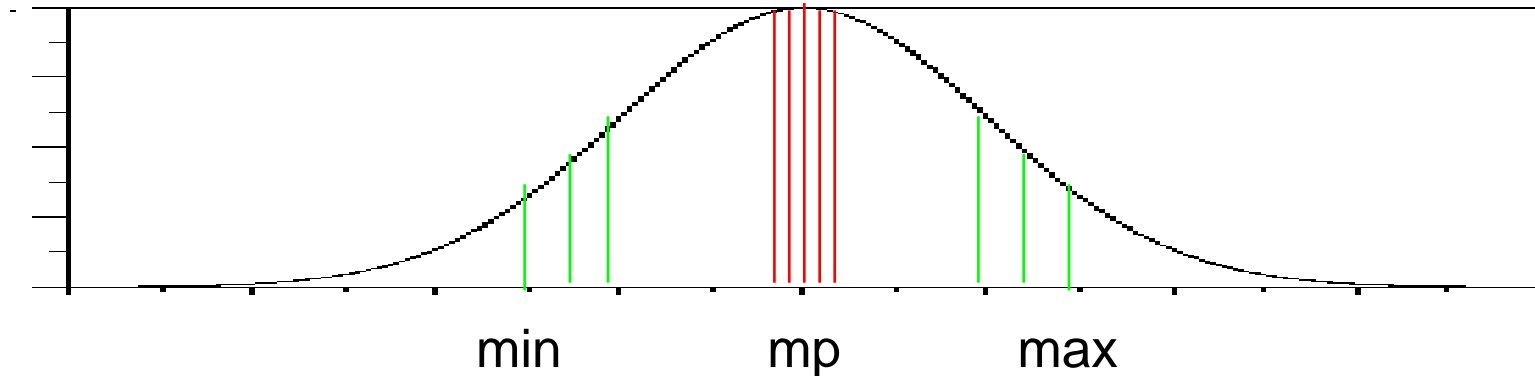
So, when the spacing of one variable is given, the spacings of other variables can be determined based on the above relation.

- For significant-effect variables, take small spacing, so they have more elements in LUT.
- For small-effect variables (almost like constants), use large spacing, so they have less elements.

## Question 6

- In a LUT, should the domain of a variable be equally spaced or not?

# *Variant spacing within a domain of variable*



Spacing within the domain of each variable does not have to be equal.

(See next-step analyses)

## *Conclusion - significance of LUT approach*

- Computer limitation
  - Speed
  - Storage
- Model design
  - Accurate
  - Fast
  - Less space
- Many efforts for this purpose
  - LUT approach is another milestone in model design; just like
  - The spectral-grid transform technique

(Bourke et al 1977; Machenhauer 1979)