

***Integrating models and observations
Data assimilation / Inverse modeling***

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Relationship between model and observations

Forward problem

$$y = f(m)$$

f - model

m - input that model result depends on

y - simulated observations

Is y “matching” actual observations?

Inverse problem (data assimilation)

Find m such that y “matches” actual observations

Probability means: probability of occurrence of a value

Integrated over all observations

$$p_m(m) \approx \int_D [p_2(m) p_2(y) p_1(y/m)] dy$$

Probability of m after - posterior

Probability of m before - prior

Includes how much is known about m before using observations

Probability of observations

Includes observation values and observation errors

Probability of modeled observations

Includes model result and model errors

Examples to illustrate not to prove inverse theory

- **The theory is already proven**
- **In practical data assimilation (examples from O'Neill) the theory is applied approximately, within practical limits: resolution, data volume etc**

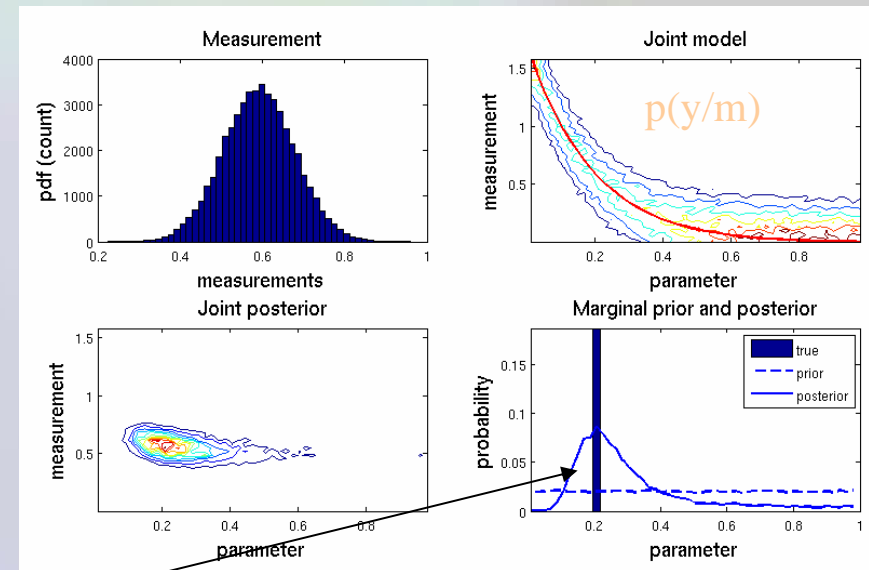
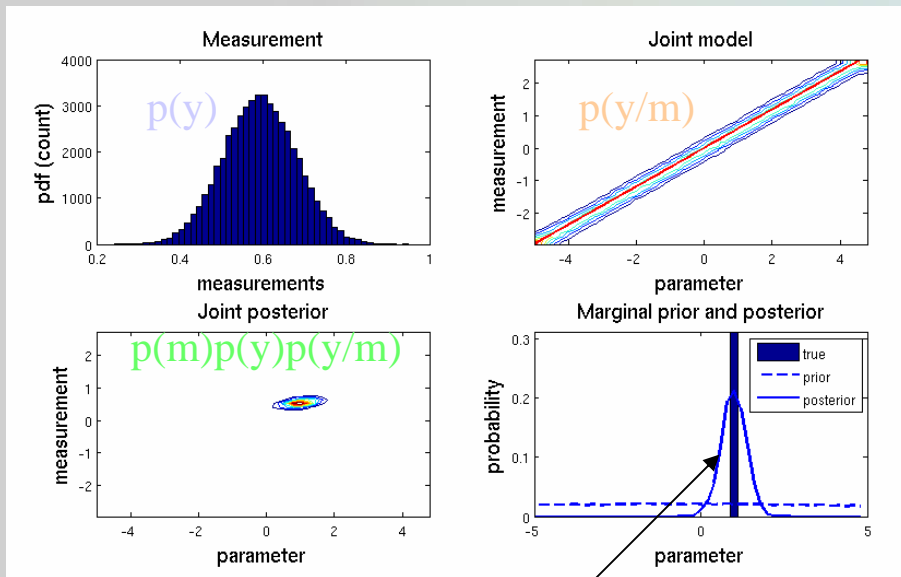
Example 1: Damped oscillations model

Observation is oscillation amplitude

$$\chi = A_1 e^{(-\lambda+\eta)\tau} + A_2 e^{(-\lambda-\eta)\tau}$$

m is initial condition

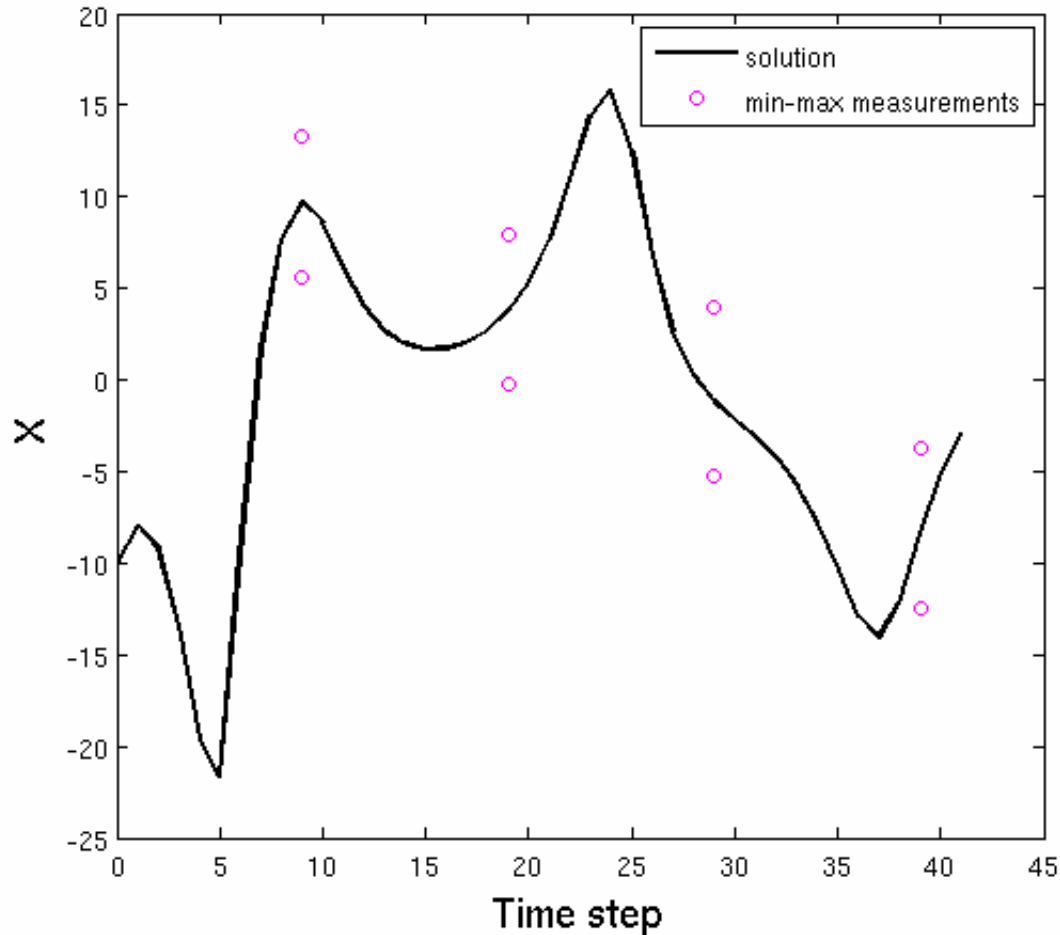
m is natural frequency



$$p_m(m) \approx \int_D [p_2(m) p_2(y) p_1(y/m)] dy$$

Example 2: Lorenz 3-component model

Observations in X component at different times

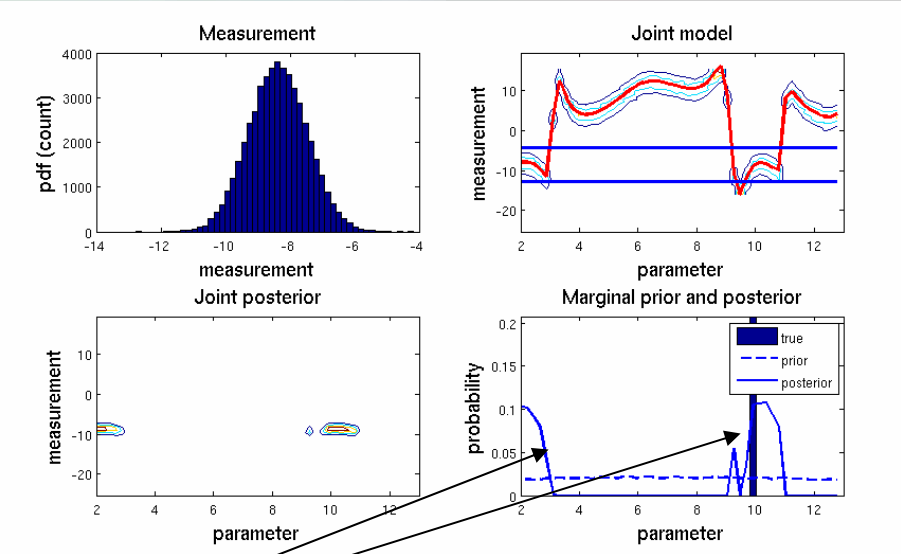
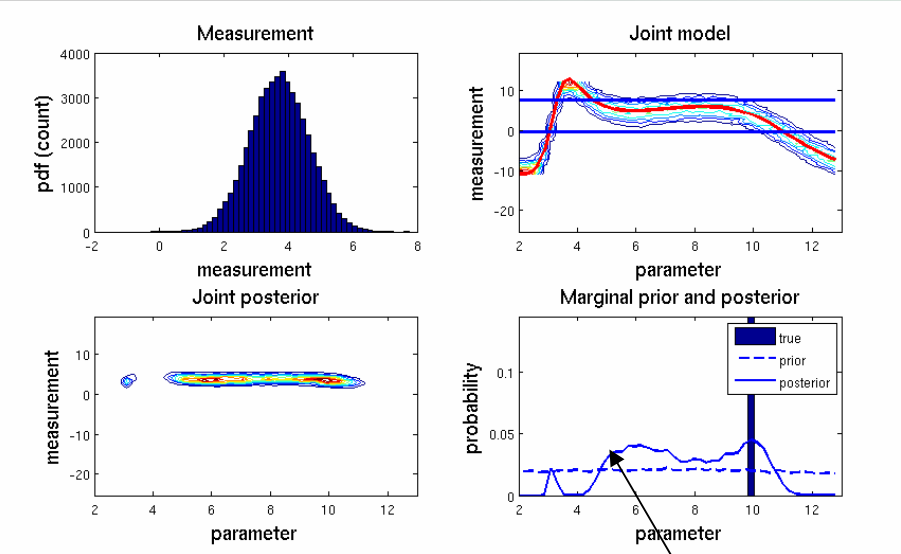


Dots represent min and max values in the observation range for each observation time

Observations every 10 time steps

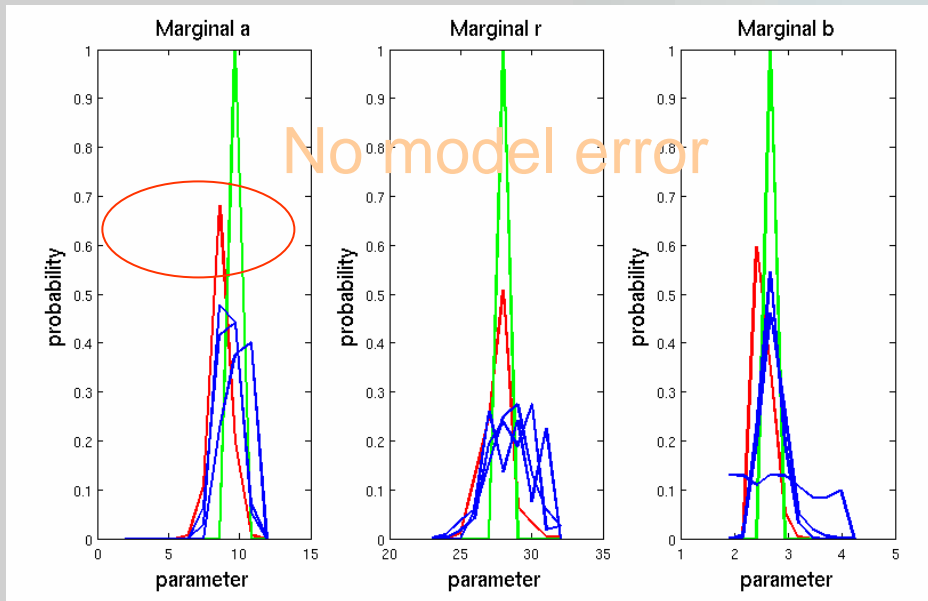
m is vector of model coefficients

Example of 2 different observation times



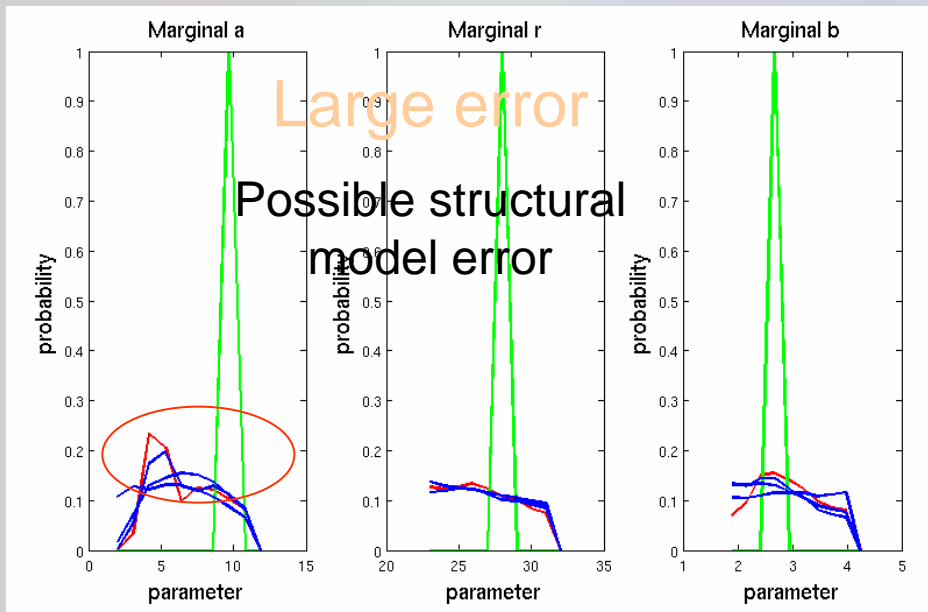
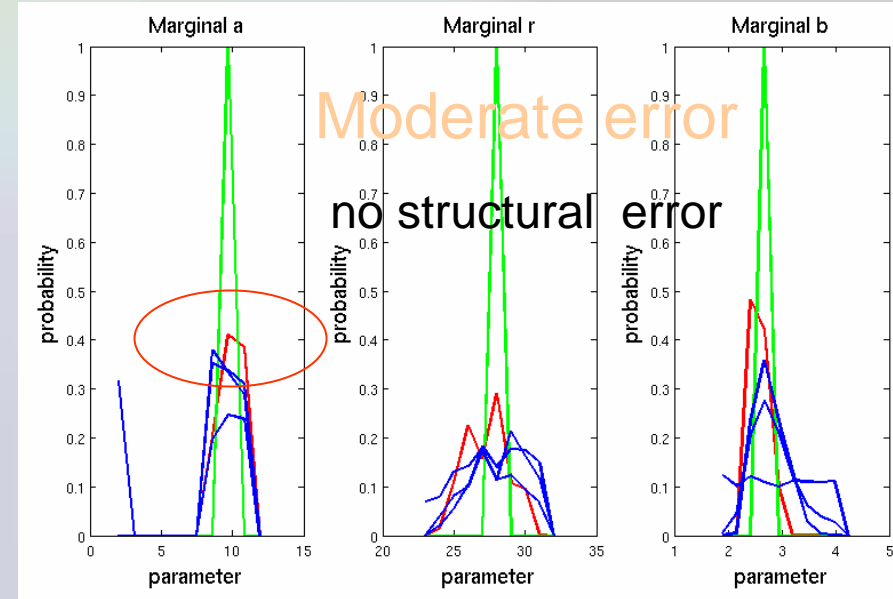
Complex $p(m)$ for each observation individually

Inverse result for all parameters in the model and all observations combined



Green is true value

Red is final result of assimilation



Summary

- **Excellent results without model error**
- **Good results with non-structural model error**
- **Non-informative results with large model error**

Could models be improved by data assimilation?

- **New research**
 - **climate and weather/process models**