



**Fig. 4-2.** A one-dimensional schematic illustration of a situation where (a)  $\overline{\phi''} \neq 0$  and (b)  $\overline{\phi''} \approx 0$ . The averaging length is illustrated by the interval  $\Delta x$  drawn on the figure ( $\overline{\phi} = \frac{1}{10} \int_{x_i}^{x_i+10} \phi dx$ ). Since  $\phi = \overline{\phi} + \phi''$ ,  $\overline{\phi''} = 0$  only if  $\overline{\phi} = \overline{\phi}$ .

Even with the simplifications, Eq. (4-7) contains two additional terms not found in Eq. (2-45) that involve the correlation between the subgrid-scale variables. The second of these terms,  $\overline{\alpha'' \partial p'' / \partial x_i}$ , could be eliminated if the assumption is made in Eq. (4-4) that  $|\alpha''| / \bar{\alpha} \approx |\alpha''| / \alpha_0 \ll 1$  [In Section 3.1,  $\alpha_0$  was defined as a synoptic-scale specific volume in the derivation of the approximate forms of the conservation-of-mass relation. The mathematical definition of this synoptic scale is given by Eq. (4-12).]

With this requirement on specific volume and the assumption that Eq. (3-11) can be written as

$$\frac{\partial}{\partial x_j} \rho_0 u_j \approx \frac{\partial}{\partial x_j} \bar{\rho} u_j \quad \left( \text{since } \frac{|\alpha''|}{\bar{\alpha}} \approx \frac{|\alpha''|}{\alpha_0} \ll 1 \right), \quad (4-9)$$

using the simplifying assumptions given by Eq. (4-8), Eq. (4-5) can be written as

$$\bar{\rho} \frac{\partial \bar{u}_i}{\partial t} = - \frac{\partial}{\partial x_j} \bar{\rho} \bar{u}_j \bar{u}_i - \frac{\partial}{\partial x_j} \overline{\bar{\rho} u_j'' u_i''} - \frac{\partial \bar{\rho}}{\partial x_i} - \bar{\rho} g \delta_{i3} - 2 \epsilon_{ijk} \Omega_j \bar{u}_k \bar{\rho}, \quad (4-10)$$

where, since  $|\alpha''| / \bar{\alpha} \ll 1$ , the pressure gradient term is represented by  $\bar{\alpha} \partial \bar{p} / \partial x$ . The remaining *subgrid-scale correlation term*,  $\overline{\bar{\rho} u_j'' u_i''}$ , represents the contributions of the smaller scales on the resolvable grid scale resulting from fluctuating velocity components and is in general very important in all aspects of dynamic

$$\bar{\rho} \frac{\partial \bar{u}_i}{\partial t} = - \frac{\partial}{\partial x_j} \bar{\rho} \bar{u}_j \bar{u}_i - \frac{\partial}{\partial x_j} \overline{\rho u_j'' u_i''} - \frac{\partial \bar{p}}{\partial x_i} - \bar{\rho} g \delta_{i3} - 2 \epsilon_{ijk} \Omega_j \bar{u}_k \bar{\rho}, \quad (4.10)$$

CORRECTION

