Relating seasonal velocity variations in the Greenland Ice Sheet to a 1D hydrology model based on alpine theory

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ABSTRACT

Nine years of uncorrected *in situ* global positioning system data demonstrates that the Sermeq Avannarleq flowline, in Western Greenland, experiences an annual velocity cycle. We develop a 1D hydrology model to demonstrate that, akin to an alpine glacier, this flowline exhibits a relation between basal sliding velocity and the rate of change in water storage (*dS/dt*). The model uses observed surface and bedrock topographies and is forced with observed surface ablation values. Englacial drainage occurs through conduits of prescribed volume and activity. Under a reasonable parameterization, the model achieves an equilibrium annual hydrology cycle. The modeled annual *dS/dt* cycle is similar to the observed surface velocity history. Thus, we speculate that variations in basal sliding velocity are modulated by variations in *dS/dt* along the flowline.

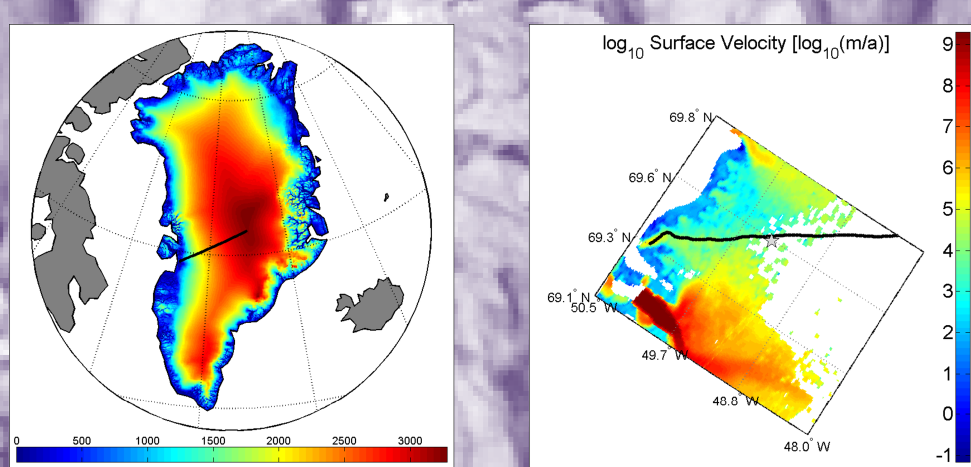

Figure 1 – Left: *The Sermeq Avannarleq flowline overlaid on observed ice surface elevation (*Scambos and Haran, *2002).* Right: *The terminal portion of the flowline, just north of Jakobshavn Isbrae, overlaid on inferred ice surface velocity (*Rignot and Kanagaratnam, *2006). The location of Swiss Camp is shown with a star.*

EMPIRICAL MOTIVATION

Recent interferometric synthetic aperture radar measurements suggest that an annual basal sliding cycle is spatially widespread in the marginal ice of Western Greenland (*Joughin et al.*, 2008). CU-ETH ("Swiss") Camp is located on an ~ 530 km long flowline which extends from the main ice divide of the Greenland Ice Sheet to the terminus of Sermeq Avannarleq ("Dead") Glacier (Figure 1). Nine years (2000/09) of uncorrected *in situ* global positioning system (GPS) data demonstrates that Swiss Camp experiences an annual velocity cycle (Figure 2). This cycle is comprised of four phases: (I) a spring speedup event, (II) a summer slowdown event, (III) a fall recovery event (in which velocities return to winter mean velocity from fall minimum velocity) and (IV) sustained winter speeds. Recent advances in alpine glaciohydrology (i.e. *Bartholomaus et al.*, 2008) suggest that changes in basal sliding velocity are closely associated with changes in the rate of water storage (*dS/dt*). We generate a 1D (flowline) hydrology model, predicated on alpine glaciohydrology theory, to investigate the annual velocity cycle at Swiss Camp in the context of changes in the rate of water storage.

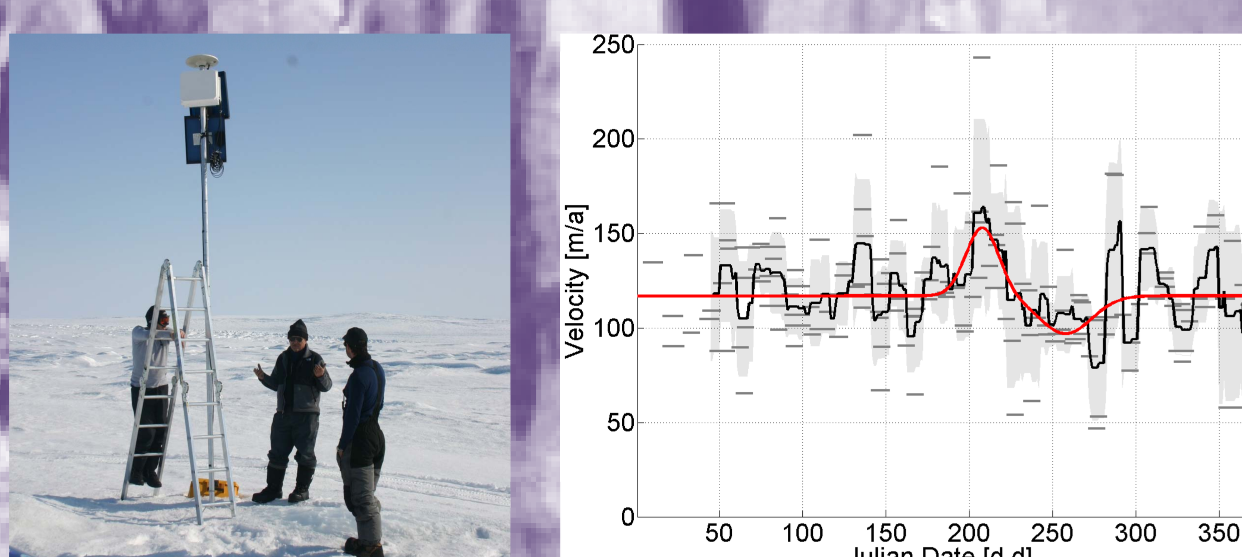

Figure 2 – Left: *Maintaining a GPS station.* Right: *Nine years of surface velocity versus Julian Date at Swiss Camp. Mean (black line) and standard deviation (grey shading) values are shown when > 3 years of data are available. The red line denotes a modeled annual velocity cycle (*Colgan et al., *2009*).

1D HYDROLOGY MODEL

We apply a 1D hydrology model, similar to that of *Flowers et al.* (2004), to the terminal portion of the Sermeq Avannarleq flowline. This model calculates the rate of change in englacial water height (dh_e/dt) due to four terms (Equation 1): surface meltwater input across a unit width (*wi*), horizontal englacial water flux variations (*dQ/dx*), internal meltwater generation from viscous melt (m/ρ_w), and changes in subglacial conduit (and cavity) volume through time (dS_c/dt). (dh_e/dt is directly proportional to *dS/dt* ; Equation 1). We assume a uniform ice porosity (*Φ*) of 1 % along the flowline.

$$\frac{dS}{dt} = w\phi\frac{dh_e}{dt} = wi - \frac{dQ}{dx} + \frac{\dot{m}}{\rho_w} - \frac{dS_c}{dt} \qquad \text{Equation 1}$$

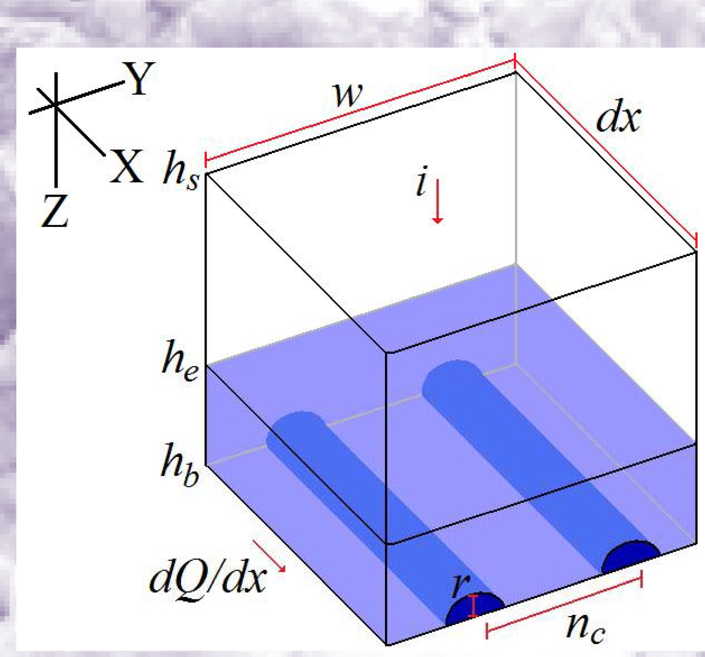

Figure 3 – *Graphical overview of the 1D hydrology model framework, with a full list of constant (constant through time and space), prescribed (constant through time), and transient model variables.*

	Constant Variables	
ϕ	ice porosity (0.01)	[ ]
ρ_i	density of ice (917)	[kg/m³]
ρ_w	density of water (1000)	[kg/m³]
dt	time-step (1)	[d]
dx	distance-step (500)	[m]
f	form factor (0.05)	[ ]
g	gravitational acceleration (9.81)	[m²/s]
n	Glen Number (3)	[ ]
n_c	conduits per unit width (0.01)	[/m]
w	unit width (100)	[m]
	Prescribed Variables	
A	Flow Law Parameter	[/Pa³/a]
h_b	bedrock elevation	[m]
h_s	ice surface elevation	[m]
i	surface meltwater input	[m/d]
P_i	ice pressure	[Pa]
r	conduit radius	[m]
S_c	conduit storage volume	[m³]
	Transient Variables	
A_c	conduit wet cross-sectional area	[m²]
D_{hc}	conduit effective hydraulic diameter	[m]
h_e	englacial water height	[m]
P_w	water pressure	[Pa]
Q	conduit discharge	[m³/d]

INPUT DATASETS

We run the model with observed ice surface (h_s) and bedrock (h_b) topographies (*Bamber et al.*, 2001; *Scambos and Haran*, 2002; *Plummer et al.*, 2008). The bedrock topography is slightly altered between km 490 and 510 to allow englacial water to flow through a subglacial mountain range (Figure 6). The boundary conditions for englacial water table height (h_e) are prescribed as sea level elevation at the glacier terminus (i.e. h_e = 0) and bedrock elevation (i.e. zero water table thickness) inland of km 450, where cold-based conditions exist (*Phillips et al.*, 2009). We use modeled mean net surface balance (*Box et al.*, 2004) and mean accumulation (*Burgess et al.*, *in press*) rates to calculate the mean ablation rate along the flowline over the 1991 to 2000 period. We distribute the mean ablation rate across the observed melt season with a first-order treatment, whereby the ablation rate increases linearly from zero at the onset of melt to a peak mid-season, and then decreases linearly back to zero at the cessation of melt (Figure 4).

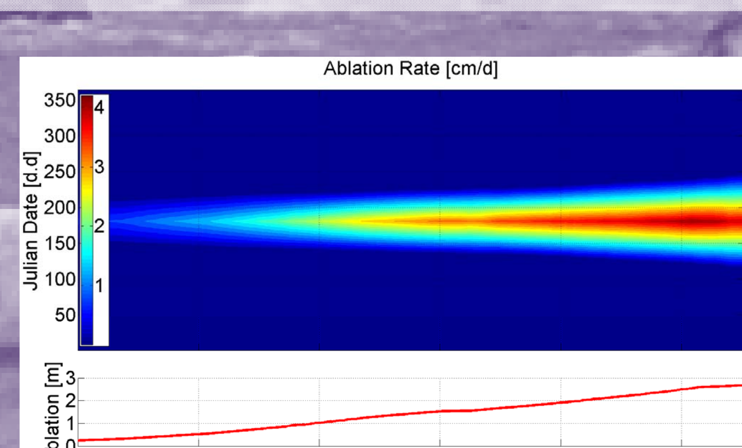

Figure 4 – *Ablation distributed in a first-order fashion across the melt season yields the surface meltwater input term (ablation rate: "i") which drives the model (Equation 1).*

EN-/SUBGLACIAL CONDUIT DISCHARGE

Horizontal englacial water flux variations (*dQ/dx*) are calculated according to non-uniform flow, whereby discharge (*Q*) is dependent on the englacial water height gradient (dh_e/dx; Equation 2). We use a semi-circular conduit geometry to calculate the conduit wet cross-sectional area (A_c) and effective hydraulic diameter (D_{hc}), based on conduit radius (*r*), at each node. This allows a smooth transition between "open" and "closed" channel flow under the conditions of (h_b+*r*) > h_e and (h_b+*r*) < h_e respectively. We assume a uniform distribution of conduits per unit width (n_c) of 0.01/m.

$$Q = \sqrt{\frac{8g}{f}} \cdot A_c \cdot D_{hc}^{1/2} \cdot \left(\frac{dh_e}{dx}\right)^{1/2} \cdot n_c w \qquad \text{Equation 2}$$

$$\frac{dS_c}{dt} = 2AS_c\left(\frac{|P_i - P_w|}{n}\right)^n - \frac{\dot{m}}{\rho_i} \qquad \text{Equation 3}$$

Presently, we simply prescribe conduit volumes (S_c) in an attempt to mimic the annual cycle. In this prescription, conduits are "closed" everywhere along the flowline during the winter (i.e. r_{min} = 0.2 m). Conduits "open" in the summer once the cumulative surface melt input at a given node exceeds 1 mWE. Conduits remain open until 14 days after the cessation of melt at given node. The conduit radius is assumed to linearly decrease from 2 m at the terminus to r_{min} at the onset of cold-based conditions at km 450 (*Phillips et al.*, 2009).

Future model versions will include an independent, physically-based, temporally-evolving subglacial conduit storage volume term (dS_c/dt). This will accommodate conduit volume using two terms: the volume created by viscous melt (m/ρ_i), and the volume change due to internal deformation. Internal deformation, which is the result of differences between ice and water overburden pressures (P_i - P_w), can either increase or decrease conduit volumes (Equation 3). The deformation term is highly non-linear, being very sensitive to both the Flow Law Parameter (*A*), and the Glen Number (*n*).

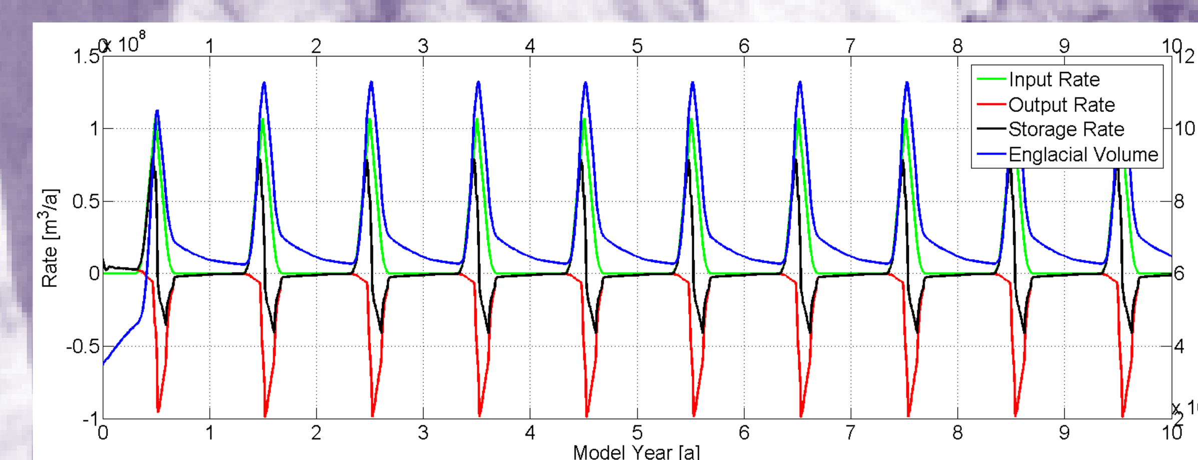

Figure 5 – *Rates of total glacier water input, output and storage, and volume during a 10-year spin-up.*

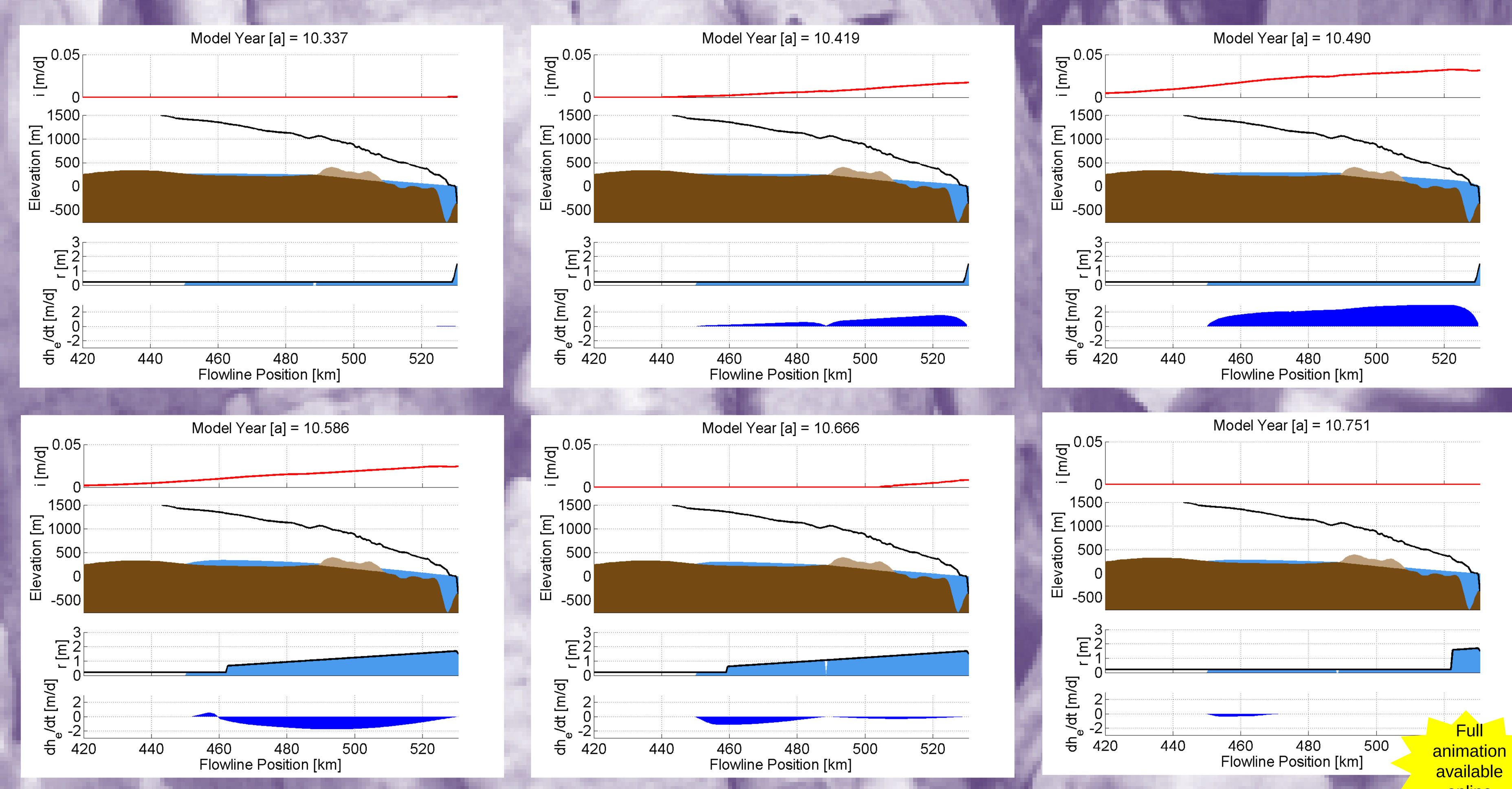

Figure 6 – *Selected model time-steps (1 May, 1 Jun, 1 Jul, 1 Aug, 1 Sep and 1 Oct). Subplot top to bottom: surface meltwater input (i), bedrock (h_b), ice (h_s) and englacial water (h_e) elevations, conduit radius (r) and "fullness", and local rate of change in englacial water height (dh_e/dt). The observed bedrock topography has been slightly altered between km 490 and 510 to allow englacial water to flow through a subglacial mountain range. Full 1-year animation available at* http://cires.colorado.edu/~colganw.

DISCUSSION

A 10-year spin-up (*dt* = 1 day) demonstrates that the model reaches equilibrium after ~ 2 years (Figure 5). Once in equilibrium, the water input and output rates, and englacial water volume exhibit steady-state annual cycles. This (untuned) model, with prescribed conduit volumes, produces an annual dh_e/dt history very similar to the observed basal sliding history at Swiss Camp (Figures 6 and 7). An important difference between these two histories is the date of summer maximum. Observed basal sliding velocities peak around JD 208, while dh_e/dt values peak around JD 185. This discrepancy is likely due to the binary treatment of conduit volume (as either "open" or "closed"). While this binary treatment contains a good approximation for the date of englacial drainage initiation, it implies an immediate establishment of efficient drainage at this date. This simplification effectively replaces a period of conduit growth (i.e. positive dh_e/dt values), with a period of efficient drainage (i.e. negative dh_e/dt values). The dh_e/dt peak will likely occur later in the melt season once the temporally-evolving conduit volume term (Equation 3) is incorporated.

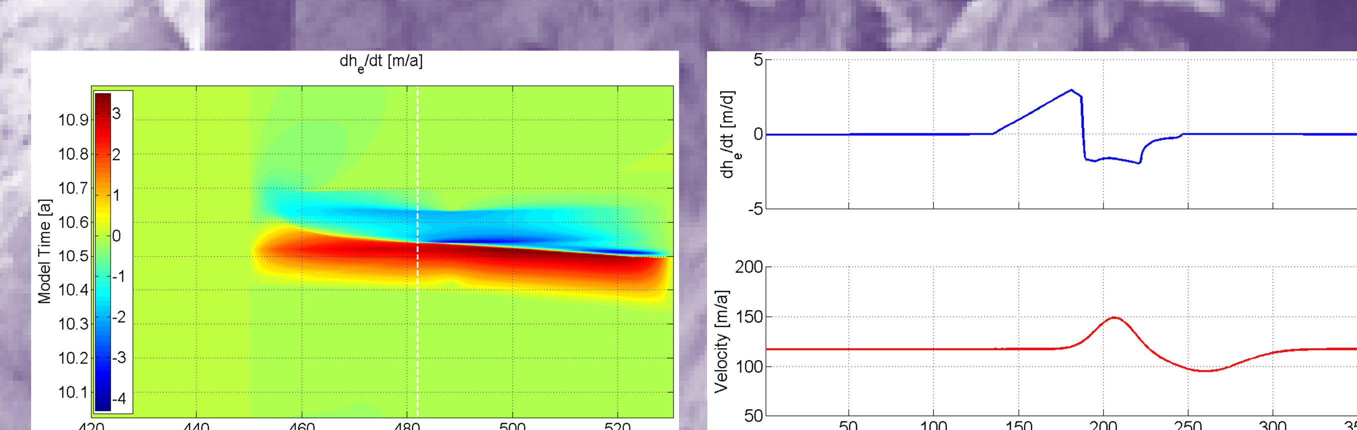

Figure 7 – Left: *Rate of change of englacial water height (dh_e/dt) during model year 10. Position of Swiss Camp is delineated with a dashed white line.* Right: *Model year 10 annual dh_e/dt history at Swiss Camp versus observed annual velocity history.*

Similar to the annual basal sliding cycle, the annual dh_e/dt cycle exhibits four phases: Phase I ("increasing storage") during the spring, when surface meltwater inputs exceed the transmission capacity of the "closed" subglacial system; Phase II ("decreasing storage") during the summer, when the transmission capacity of "open" subglacial system allows efficient drainage of surface meltwater inputs; Phase III ("storage recovery") at the end of the melt season when dh_e/dt values recover to zero; and Phase IV ("winter steady-state") when there is no change in storage rate outside the melt season. The similarities between the modeled dh_e/dt and observed basal sliding histories suggests that this Greenland Ice Sheet flowline may respond to surface meltwater input in a manner akin to alpine glaciers, whereby basal sliding variations are proportional to dh_e/dt variations.

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