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The Upper Atmosphere

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II.3.1.4 Three-Dimensional Ray Tracing in the Atmosphere

1 Introduction

Ray tracing is a practical method for calculating the ray theory or WKB (Wentzel 1926; Kramers 1926; Brillouin 1926) approximation for the propagation of waves in a specified medium (Sect. 8, below). Here we consider the propagation of electromagnetic waves in the atmosphere, although nearly all of the results are directly transferable to other kinds of waves propagating in other media. In three-dimensional ray tracing, the medium varies in three dimensions, and therefore, the wave will follow a three-dimensional path rather than being confined to a plane.

Except for scintillations, the refraction of electromagnetic waves by the neutral upper atmosphere is negligible. Because ray tracing is of limited use in calculating scintillation, we restrict the discussion to the propagation of radio waves in the ionized atmosphere (ionosphere).

The general problem of calculating radio wave propagation in the ionosphere is as follows: we have a radio transmitter, either on the ground or elevated (as in an airplane or satellite); emitting radio waves at some frequency, either continuous waves (cw) or pulsed waves; and a receiver, either on the ground or elevated. For given ionospheric conditions, represented by a refractive index [Chap. II. 3.1.3 Eq. (4.8)] that varies with location and direction of the radio wave, we would like to estimate the following quantities:

1. The signal strength at the receiver.
2. The phase of the radio wave at the receiver.
3. The travel time for a pulse to propagate from the transmitter to the receiver.
4. The direction(s) of arrival of the wave at the receiver.
5. The polarization of the wave at the transmitter and receiver.
6. The wave form (pulse shape) distortion by dispersion.
7. The statistical behavior (e.g. spatial and temporal coherence) of the signal if the medium can be described usefully only in a statistical sense.

Usually, ray tracing is not useful for correctly dealing with the statistical nature of the medium, but it can estimate the first six of the above seven quantities. It is useful to divide the ray tracing method into ten steps:

1. Calculate all of the raypaths from the transmitter to the receiver.
2. Calculate the pulse travel time of the wave for propagation along each of these raypaths.
3. Calculate the attenuation of the wave (absorption of energy by the medium) for propagation along each of these raypaths.
4. Calculate the phase of the radio wave for propagation along each of these raypaths.
5. Calculate the effect of focussing or defocussing (convergence or divergence of adjacent rays) on the amplitude of the wave for each raypath (valid if the receiver is not too near a caustic).

6. Subtract 90° from the phase for each surface caustic through which the ray has passed (valid if the receiver is not too near a caustic).
7. Adjust the amplitude and phase of the radio wave for each raypath according to any ground reflections that may have occurred.
8. Calculate the coupling of the wave to the antenna at the transmitter and the receiver, including the effects of the antenna pattern factor and polarization.
9. Combine the amplitudes of the radio waves for the various raypaths coherently (if the signals are correlated) or incoherently (if the signals are not correlated). Thus, we have the possibility of phase interference between radio waves that have traveled separate paths.
10. For pulse transmission, combine (coherently or incoherently the amplitudes of the signals from the various paths only for pulses that overlap in time, and then, as a function of time for the overlapping time period. (The pulse shape can be calculated by a perturbation method, or by Fourier analyzing the shape of the transmitted pulse, propagating each spectral component separately, and combining the components.)

The first of the ten steps is the most difficult, and is the main task of ray tracing. It is well known, that there is, in general, no closed-form solution to the problem of finding all of the raypaths connecting a specified transmitter and receiver, although there are a few simple cases (such as propagation in a homogeneous medium) where it can be done.

Although a raypath is not uniquely determined by specifying the transmitter and receiver location, it is uniquely determined by the transmitter location and the direction of transmission. Therefore, we divide the task of calculating the raypaths that connect a specified transmitter and receiver into two parts.

First, we calculate a raypath for a specified transmitter location, launch direction, and specified medium. Section 2 discusses how this is done with a ray tracing computer programme. Central to such raypath calculations is the question of how to specify the medium, which is discussed in Sect. 3.

Second, we find out which raypaths actually arrive at the receiver. (such raypaths are sometimes called eigenrays.) Section 4 discusses how this process, called eigenray determination, is done. (Of course, eigenray determination and raypath calculation are approximate in any computer calculation. It is always necessary to verify that the raypaths are determined sufficiently accurately for the application.)

There is a variational method for calculating raypaths that is an alternative to ray tracing (Chander 1975; Julian and Gubbins 1977; Červený and Hron 1980; Pereyra et al 1980), but we do not deal with that method here.

Step 2 of the ray tracing method (calculation of pulse travel time) is already taken care of if we use the four-dimensional form of Hamilton's equations. That is, a path in four dimensions already includes the beginning and ending time of the path.

The amplitude and phase of the wave (step 3 and 4) must be calculated by the ray tracing program for each raypath. Section 5 discusses this in detail.

Steps 5–8 require making adjustments to the amplitude and phase of the wave for propagation along each of the raypaths. Section 6 discusses these adjustments.

Steps 9 and 10 are concerned with combining the signals for the waves that have propagated along the various raypaths, both for cw and pulse propagation. Section 7 discusses how these signals should be combined.

Although the ray theory (WKB) approximation is not always valid, raypath calculations with a ray tracing programme can nearly always give insight into the propagation. Section 8 discusses the validity of ray theory.

Section 9 gives a short description of the characteristics of a ray tracing programme as an example.

2 Method for Calculating Raypaths

For a specified medium, a raypath is determined by the transmitter location and the direction of transmission. In regions where the refractive index of the medium varies continuously with position and time, the ray evolves according to Hamilton's equations (e.g., Lighthill 1978, p. 319; Budden 1985, p. 404): [see Sect. 8 for the justification of Hamilton's equations in terms of the WKB approximation for calculating raypaths. Arnold (1978) gives an intuitive argument in terms of Huygens' principle.]

$$dq_i/d\tau = \partial H(q_i, p_i)/\partial p_i \quad (i = 0\text{–}3) , \tag{2.1a}$$

$$dp_i/d\tau = -\partial H(q_i, p_i)/\partial q_i \quad (i = 0\text{–}3) , \tag{2.1b}$$

where q_i are generalized coordinates, p_i are generalized momenta canonical to the coordinates, and $H(q_i, p_i)$ is a Hamiltonian that depends on position and wave direction, as expressed by the coordinates and momenta. Equation (2.1) expresses how the position and wave direction change along a ray as it progresses. The parameter τ varies monotonically along the raypath. Its physical interpretation depends on the form of the Hamiltonian.

Hamilton's equations in Eq. (2.1) are four-dimensional, to include the effect of a time-varying medium (Lighthill 1965). A Hamiltonian that depends on four coordinates (including time) rather than three spatial coordinates only is referred to by Misner et al. (1973) as a "super-Hamiltonian." The main effect of including the time coordinate in Hamilton's equations is to add the calculation of the frequency shift (Doppler shift) of the wave for propagation in a time-varying medium. [See Bennett (1967) for an alternate derivation of the Doppler-shift formula.] This can be seen more explicitly if we express Eq. (2.1) in Cartesian coordinates (Weinberg 1962):

$$dt/d\tau = -\partial H(t, x_i, \omega, k_i)/\partial \omega , \tag{2.2a}$$

$$dx_i/d\tau = \partial H(t, x_i, \omega, k_i)/\partial k_i \quad (i = 1\text{–}3) , \tag{2.2b}$$

$$d\omega/d\tau = \partial H(t, x_i, \omega, k_i)/\partial t \ , \tag{2.2c}$$

$$dk_i/d\tau = -\partial H(t, x_i, \omega, k_i)/\partial x_i \quad (i = 1\text{–}3) \ , \tag{2.2d}$$

where t is the pulse travel time (group time), ω is the wave frequency (which can vary along the path if the medium varies with time), x_i are the cartesian coordinates of the ray point, k_i are the cartesian components of the wave vector $\boldsymbol{k}$, and ω is the wave frequency, which will vary along the raypath if the medium is time varying. The direction of $\boldsymbol{k}$ is normal to the wave front, and the magnitude of the wave vector is

$$k = \sqrt{(k_1{}^2 + k_2{}^2 + k_3{}^2)} = \omega n/c = 2\pi/\lambda \ , \tag{2.3}$$

where n is the refractive index, c is the free-space speed of light, and λ is the radio wavelength. Notice that the four-dimensional form of Hamilton's equations automatically yields the pulse travel time through Eq. (2.2a), without having to bring in the notion of group velocity, because it is equivalent to the travel time given in the usual way by group velocity (Lighthill 1965, 1978). The four-dimensional form of Hamilton's equations also automatically gives the frequency shift of the wave [Eq. (2.2c)] for propagation through a time-varying medium.

The Hamiltonian can take various forms. One form is

$$H(t, x_i, \omega, k_i) = f[c^2k^2 - \omega^2 n^2(t, x_i, \omega, k_i)] \ , \tag{2.4}$$

where f is any reasonable function, n is the refractive index, and c is the free-space speed of light. Section 3 considers other forms for the Hamiltonian.

The refractive index depends only on the direction of $\boldsymbol{k}$, and not on it's magnitude, as is generally apparent in the explicit functional dependence of n on $\boldsymbol{k}$ [Chap. II. 3.1.3 Eq. (4.8)]. However, when taking derivatives of n with respect to one of the components of $\boldsymbol{k}$ in Hamilton's equations (2.1) or (2.2), one must be careful to use the constraint that the other two components of $\boldsymbol{k}$ remain constant rather than the constraint that the magnitude of $\boldsymbol{k}$ remain constant. It is usually easier to avoid errors in calculating the formulae for these derivatives if n is written explicitly in terms of the components of $\boldsymbol{k}$ rather than in terms of the components of a unit vector in the $\boldsymbol{k}$ direction or in terms of the direction of $\boldsymbol{k}$.

Notice that if f is a homogeneous function, then

$$H(t, x_i, \omega, k_i) = f[c^2k^2 - \omega^2 n^2(t, x_i, \omega, k_i)] = 0 \tag{2.5}$$

is an expression of the dispersion relation for the wave.

Generally, the system of equations (2.2) can be integrated by standard computer numerical integration algorithms. For the partial derivatives of the Hamiltonian to be numerically well defined, the Hamiltonian (and therefore the refractive index) must be a continuous function of its arguments. For that integration to be efficient, the derivatives must also be continuous functions of the arguments (e.g. the gradient of refractive index must be a continuous function of position).

Examples of Hamiltonian ray tracing programmes are described by Georges (1971), Jones and Stephenson (1975), and Jones et al. (1982, 1986a, b). Examples of the applications of these ray tracing programmes are given by Georges and Stephenson (1968), Georges (1970, 1972), Jones et al. (1984), and Georges et al. (1986).

At an interface where the refractive index changes discontinuously, Hamilton's equations cannot be used to calculate the amount of bending at the interface, but snell's law,

$$n_1 \sin \Theta_1 = n_2 \sin \Theta_2 \ , \tag{2.6}$$

must be used instead. In Eq. (2.6), n_1 and n_2 are the refractive indexes in the medium of the incident wave and transmitted wave, and Θ_1 and Θ_2 are the angle of incidence and the angle of transmission into the second medium. In an anisotropic medium (such as the ionosphere), where the refractive index depends not only on position but also on direction of the wave, Snell's law [Eq. (2.6)] must be solved iteratively with the dispersion relation (2.5) for Θ_2, the angle of transmission into the second medium, and for n_2, the refractive index of the second medium.

Snell's law (2.6) is equivalent to

$$\boldsymbol{k}_{\|1} = \boldsymbol{k}_{\|2} \ , \tag{2.7}$$

where $\boldsymbol{k}_{\|1}$ and $\boldsymbol{k}_{\|2}$ are the components of the wave vector parallel to the interface of discontinuity of refractive index. The normal component of the wave vector,, $k_{\perp 2}$, is found from the dispersion relation [Eq. (2.5)].

A discontinuity in refractive index also causes partial reflection of the wave. In an anisotropic medium (such as the ionosphere), the angle of reflection does not equal the angle of incidence. Instead, Snell's law again applies, either in the form of Eq. (2.6) or Eq. (2.7), where for reflection, the subscripts 1 and 2 refer to incident and reflected. The refractive indexes n_1 and n_2 are unequal for the same medium, because the wave directions are different for the incident and reflected wave.

For practical calculations of radio wave propagation in the ionosphere, however, the only discontinuity in refractive index is at the ground, where the refractive index is isotropic, and therefore the angle of reflection equals the angle of incidence. Also, we can usually neglect any part of the radio wave that is transmitted into the ground (unless there is a radio receiver underground, but very close to the surface).

Therefore, in practice, we can neglect the complications of Snell's law for radio wave propagation in the ionosphere. However, there are some ray tracing programmes that approximate the continuous variation of the ionospheric refractive index by using homogeneous slabs, and apply Snell's law at the boundaries. This can be practical when the anisotropy of the ionosphere can be neglected (Croft and Gregory 1963).

Earth-centred spherical-polar coordinates are more practical than cartesian coordinates for calculating long-distance radio wave propagation. Jones and Stephenson (1975) give Hamilton's equations in spherical-polar coordinates.

3 Representation of the Propagation Medium

There are two main issues with regard to how to represent the propagation medium. The first is whether to represent it as a continuously varying medium in regions where it is truly continuous and as discontinuous only where it is truly discontinuous or to approximate the medium by a set of homogeneous regions. The second is how to express the Hamiltonian.

If the medium is approximated by a set of homogeneous regions, then the raypath must be calculated using Snell's law rather than Hamilton's equations. Usually, Snell's law calculations are practical only when the anisotropy can be neglected and when the raypath is confined to two dimensions. If the medium is anisotropic, applying Snell's law requires iteration because the refractive index depends not only on position, but also on the wave direction. (Thus, for an anisotropic medium, the "homogeneous" regions will be homogeneous only for a single ray direction.) Further, applying Snell's law in three dimensions could be tedious. See Croft and Gregory (1963) for an example of a Snell's law ray tracing program in two dimensions that neglects the anisotropy of the ionosphere.

Equation (2.4) gives one class of Hamiltonians that are expressed in terms of refractive index. One example is for

$$f(x) = x \,. \tag{3.1}$$

That gives

$$H(t, x_i, \omega, k_i) = c^2k^2 - \omega^2 n^2(t, x_i, \omega, k_i) \tag{3.2}$$

for the Hamiltonian, and because Eq. (3.1) is a homogeneous function,

$$H(t, x_i, \omega, k_i) = c^2k^2 - \omega^2 n^2(t, x_i, \omega, k_i) = 0 \tag{3.3}$$

for the dispersion relation.

Usually, the refractive index is a complex function, with the real part giving the refractive properties of the medium, and the imaginary part giving the dissipation of the wave due to energy being absorbed by the medium. For HF (3–30 MHz) and higher frequencies, we can usually neglect the effect of dissipation on the refraction of the radio waves. In that case, we can use

$$f(x) = \mathrm{Re}\,(x) \tag{3.4}$$

in Eq. (2.4) to define a Hamiltonian. However, Section 8 considers those cases where the dissipation significantly affects refraction.

There are possibilities for the Hamiltonian other than Eq. (2.4). For example, we could use

$$H(t, x_i, \omega, k_i) = f_1[1 - \omega n(t, x_i, \omega, k_i)/(ck)] \,. \tag{3.5}$$

Using

$$f_1(x) = \mathrm{Re}\,(x) \tag{3.6}$$

would be equivalent to the Hamiltonian used by Haselgrove (1954) except for numerical integration errors and the value of the independent variable. The Hamiltonian she actually used is

$$f_1(x) = 1/[1 - \mathrm{Re}\,(x)] - 1 \,. \tag{3.7}$$

Another possibility for the Hamiltonian based on the refractive index is

$$H(t, x_i, \omega, k_i) = f_2[c^2k^2/\omega^2 - n^2(t, x_i, \omega, k_i)] \,. \tag{3.8}$$

The ray tracing programme by Jones and Stephenson (1975) uses Eq. (3.8) for a Hamiltonian, with

$$f_2(x) = \mathrm{Re}\,(x)/2 \,. \tag{3.9}$$

Two common formulae express the refractive index as a function of the electron density, the Earth's magnetic field and the collision frequency, for use in the above equations: the magnetoionic formula for constant collision frequency and the Sen–Wyller formula (which allows for collisions of electrons with neutral molecules that have a distribution of kinetic energy). These are discussed in Chap. II. 3.1.3.

The magnetoionic formula for constant collision frequency has been known for many years as the Appleton–Hartree formula (Budden 1961, 1985) or by the Appleton formula (Davies 1965, 1969). Ratcliffe (1959) refers to it as "Appleton's equations." The formula was first published by Lassen (1927). Goldstein (1928) published a version without damping. Appleton (1928) first presented the formula (also without damping) at a 1927 URSI conference, and later (Appleton 1932) published it with damping. Hartree (1931) included a Lorentz polarization term with the formula for refractive index. Appleton (1932) gave the formula both with and without the Lorentz polarization term. It is generally accepted now that the Lorentz polarization term should not be included (Budden 1961, 1985; Davies 1965, 1969; Ratcliffe 1959; Rawer and Suchy 1976). The name "Appleton–Lassen formula" seems to be coming into increasing use (Rawer and Suchy 1976; Budden 1985, p. 75). Wilhelm Altar derived the formula, but his derivation has only recently been published (Gillmor 1982). See the paper by Sen and Wyller (1960) and Chapter II. 3.1.3 for additional discussion of their formula. Here, we use "magnetoionic formula" to refer to both the version with constant collision frequency or to the Sen–Wyller formula whenever there is no need to make a distinction.

There is one situation for radio wave propagation in the ionosphere in which it is not appropriate to express the Hamiltonian in terms of the refractive index because the refractive index depends not only on the local values of electron density, magnetic field, collision frequency, and wave direction, but also on the immediately preceding history of the radio wave. This occurs at a Spitze (defined by Poeverlein 1949, 1950; Davies 1965, pp. 202–204, 1969, pp. 183–187). For that situation, it is more appropriate to use for the Hamiltonian the quadratic equation in n^2 whose solution is the magnetoinoic formula for refractive index [Chap. II. 3.1.3 Eq. (4.4)]. One can also derive that quadratic equation by specializing the Booker quartic (Booker 1949) to vertical-incidence propagation.

The propagation medium is characterized not only by its refractive index, but also by certain polarization relations that give more detailed information about the local properties of the wave beyond energy flux and the wave front. In the case of electromagnetic waves, the polarization relations give the ratios of components of the electric field of the wave. The polarization relations are characteristic of each magnetoionic component (ordinary and extraordinary). See Chapter II. 3.1.3 [Eq. (4.7)] and Budden (1961, pp. 48–54; 1985, p. 70) for more details.

4 Eigenrays: the Raypaths that Connect a Specified Transmitter and Receiver

Two methods are used to find the rays that arrive at the receiver. (These rays are sometimes called eigenrays.) First is the homing method, in which the launch direction is varied (in both azimuth and elevation) until the ray arrives at the receiver. This method has the advantage of being direct but the disadvantage of not guaranteeing that all of the eigenrays have been found.

In the second method, a fan of rays (in both azimuth and elevation angles) is transmitted, and interpolation between rays that surround the receiver is used to find the eigenrays. Usually, the angle increment in the fan of rays is determined by the required accuracy and the amount of small-scale structure in the ionospheric model. This method has the advantage of giving information about all of the rays that leave the transmitter, in addition to the eigenrays. It has the further advantage that eigenrays for a different receiver location can be found without having to calculate any new raypaths. Stephenson and Georges (1969) describe a programme that uses this method.

Although there are advocates of both methods, I have always favored the latter method. In either method, it is necessary to have a criterion to estimate the amount by which the ray has missed the receiver. This could be done by estimating the distance of the raypath to the receiver at its closest approach. In practice, however, it is usually easier to specify a surface (horizontal or vertical, for example) in which the receiver lies, and then find all of the intersections of a raypath with that surface. The amount by which the ray misses the receiver is measured as a distance on this surface. For the latter eigenray-finding method, the calculated raypaths can be used to find eigenrays for all receivers that lie on the surface specified for the raypath calculation.

Once an eigenray connecting the transmitter with the receiver has been found, it is possible to calculate most of the needed information about the signal at the receiver. Once the raypath is known, the $\boldsymbol{k}$ vector at the receiver gives the direction of the normal to the wave front there.

5 Amplitude and Phase Calculation

To calculate useful estimates of the signal at the receiver, it is necessary to calculate amplitude and phase.

The signal at the receiver for propagation along one raypath is proportional to

$$\exp(i\phi), \tag{5.1}$$

where

$$\phi = \int p_i dq_i \quad (i \text{ is summed from 0 to 3}) \tag{5.2}$$

is the phase integral (Eckersley 1932; Budden 1961, 1985; Wait 1962), and gives the phase of the wave in radians. The integral is along the raypath from the transmitter to the receiver. In general, the path of integration for the phase integral can allow the coordinates of the path to take on complex values. Section 8 discusses this more fully. Here, we consider that the raypath has real coordinates.

The q_i are the generalized coordinates in Hamilton's equations, and the p_i are the canonical momenta (e.g. Lighthill 1978, p. 319; Budden 1985, p. 404). The raypath is four-dimensional, because it includes the possibility of a time-varying medium.

As is well known, a raypath has the property that the action or phase integral; [Eq. (5.2)] is stationary with respect to variation of the path, keeping the endpoints fixed. That is,

$$\delta\phi = 0\,. \tag{5.3}$$

We can write Eq. (5.2) in differential form as

$$d\phi/d\tau = \sum_i p_i \; dq_i/d\tau \quad (i \text{ summed from 0 to 3})\,. \tag{5.4}$$

Substituting Hamilton's equation (2.1a) into (5.4) gives

$$d\phi/d\tau = \sum_i p_i \; \partial H(q_i, p_i)/\partial p_i \quad (i \text{ summed from 0 to 3})\,. \tag{5.5}$$

We can see the time dependence more explicitly if we write Eqs. (5.4) and (5.5) in cartesian coordinates:

$$d\phi/d\tau = -\omega \; dt/d\tau + \sum_i k_i \; dx_i/d\tau\,, \tag{5.6a}$$

$$d\phi/d\tau = \omega \; \partial H(t, x_i, \omega, k_i)\;/\partial\omega + \sum_i k_i \; \partial H\,(t, x_i, \omega, k_i)/\partial k_i \tag{5.6b}$$

$$(i \text{ summed from 1 to 3})\,.$$

Either form of Eq. (5.6) can be used to calculate the propagation contribution to the phase of the wave for a single raypath. The phase in Eq. (5.6) includes the time contribution in a way that shows the symmetry between space and time.

For propagation through a time-independent medium, the time contribution to the phase is often left implicit.

Dissipation of the wave through absorption of energy by the medium can be taken into account by adding an imaginary part to the refractive index. (For some media, dissipation and refraction are so strongly connected that a complex refractive index results automatically from the physics of propagation, as in the case of the magnetoionic formula (see Chap. II. 3.1.3): In most cases, the effect of dissipation on the raypath can be neglected, although Section 8 discusses situations where dissipation significantly alters the raypath.

Thus, dissipation can be taken into account by first adding the appropriate imaginary part to the refractive index, and then using Eq. (2.3) to calculate the corresponding imaginary part of $\boldsymbol{k}$. Substituting a complex $\boldsymbol{k}$ into Eq. (5.6) gives a complex ϕ, whose imaginary part gives the attenuation in nepers. An equivalent formula for attenuation is

$$dA_{nepers}/d\tau = |k_{imag}/k_{real}| d\phi/d\tau = |n_{imag}/n_{real}| d\phi/d\tau \ . \tag{5.7}$$

The attenuation in decibels is Davies 1969, p. 144; Jones and Stephenson 1975, p. 8)

$$dA_{dB}/d\tau = 20 \ \log_{10} e |k_{imag}/k_{real}| d\phi/d\tau \ . \tag{5.8}$$

Usually, the imaginary part of k is much smaller than the real part, so Eq. (5.8) can be approximated by

$$dA_{dB}/d\tau = 10 \ \log_{10} e |k^2_{imag}/k^2_{real}| d\phi/d\tau \ . \tag{5.9}$$

6 Adjustments to the Amplitude and Phase

As adjacent rays converge or diverge, the amplitude of the wave will increase or decrease because the energy of the wave is concentrated in some regions while it is diminished at others. There is no exchange of energy between the wave and the medium, but energy of the wave is simply distributed from one place to another.

Lighthill (1978, Sect. 4.5) gives the general formulae. If the medium does not absorb energy from the wave, then conservation of energy implies that the energy density W and the energy flux $\boldsymbol{I} = W\boldsymbol{U}$ satisfy the continuity equation

$$dW/\partial t + \nabla \cdot \boldsymbol{I} = 0 \ , \tag{6.1}$$

where

$$\boldsymbol{U} = \partial\omega/\partial\boldsymbol{k} \tag{6.2}$$

is the group velocity. For cw propagation in a time-independent medium, the energy density of the wave may vary with position, but it is independent of time, so the first term of Eq. (6.1) is zero. In that case, the second term in Eq. (6.1) is required to be also zero. That is, there are no sources or sinks of energy flux; therefore the flux transmitted in some solid angle bounded by a narrow bundle of rays remains bounded by those same rays. The intensity of the wave is then inversely proportional to the cross-sectional area bounded by those rays. That is,

$$IA_2 = WUA_2 = \text{constant along a ray tube}\,, \tag{6.3}$$

where A_2 is the cross-sectional area of the ray tube.

These formulae can also be justified directly from the WKB approximation or from an asymptotic series approximation to solutions of the wave equation (Weinberg 1962; Felsen and Marcuvitz 1973).

There are two methods for calculating the cross-sectional area of the flux tube. The simplest is to calculate four neighbouring raypaths (usually using two different azimuth angles of transmission and two different elevation angles of transmission), and estimate the cross-sectional area bounded by the rays. If the rays are close enough together, the cross-sectional area can be accurately estimated, and will give a good estimate of the effect of focussing.

The other method calculates differential rates of divergence or convergence of rays along a single ray. This method is in principle more accurate, but in practice more complicated and more difficult to use. For a medium that varies arbitrarily in three dimensions, this calculation requires adding six more differential equations to Hamilton's equations, and requires the refractive index models to be continuous through second derivatives rather than just through first derivatives. The necessary equations were derived by C. C. Harvey in 1968, then at Cambridge University, but were never published.

For the special case in which the medium varies in only two dimensions (in height and in horizontal range from the transmitter), the focussing equations are quite simple. In that case, the ratio of the wave intensity (square of the amplitude) at the receiver to that at a small distance r_0 from the transmitter is

$$U_0 r_0^2 \cos\alpha \,/\, (Ur \sin\beta \, dr/d\alpha)\,, \tag{6.4}$$

where α is the elevation angle of transmission, r is the horizontal distance from the transmitter to the receiver, β is the elevation angle of arrival at the receiver, $dr/d\alpha$ gives the rate of change of range with elevation angle of transmission, U_0 is the magnitude of the group velocity at the transmitter, and U is the magnitude of the group velocity at the receiver. The quantity $dr/d\alpha$ can be estimated by taking the ratio of the change in range to the difference in elevation angle of transmission for two closely spaced rays.

For the more general case, in which the medium can be time varying and the transmitted wave can also be time varying, the general formula (6.1) must be used. In that case, the calculation can be done using four-dimensional flux tubes in space–time by an appropriate generalization of Eq. (6.3).

The formulas for focussing predict an infinite signal amplitude whenever adjacent rays cross, because the cross-sectional area of adjacent rays will be zero there. For example, at the skip distance for radio waves, $dr/d\alpha$ will be zero, and therefore Eq. (6.4) diverges. Such places are called caustics. Ray theory (WKB approximation) breaks down at caustics, and ray theory does not give an accurate estimate of the wave amplitude in a region around caustics. Nevertheless, raypaths can be accurately computed through caustics.

Furthermore, ray theory can be extended to give an accurate estimate of the signal strength in the region around caustics in terms of Airy functions (Budden 1961; Ludwig 1966; Kravtsov 1964a,b; White and Pedersen 1981). One result of this extension shows that there is a 90° phase retardation each time a wave passes through a surface caustic. (That is, 90° must be subtracted from the phase lag that is being accumulated along the ray.)

Usually, there is a surface caustic at each ionospheric reflection point for low-angle rays (those rays for which range decreases as the elevation angle of transmission is increased), but not for high-angle rays. Thus, low-angle rays usually need phase correction for caustics, but not high-angle rays.

Each time the ray reflects from the ground, there will be a loss in amplitude, and a phase shift. These can be expressed by a ground reflection coefficient. In general, the ground reflection coefficient may depend on the frequency of the radio wave, the properties of the ground, and the angle of incidence on the ground. Of course, the ground properties may vary with location.

The effect of ground reflections can be taken into account after the raypaths have been calculated if the location of each ground reflection and the associated angle of incidence on the ground is saved along with the raypath calculations.

The amplitude and phase of the wave at the receiver are also affected by how the wave couples to the antenna at the transmitter and receiver. There are two aspects to this coupling.

First is the pattern factors of each antenna, that is, the effectiveness of the transmitting antenna in radiating in the direction of the wave normal at the transmitter, and the effectiveness of the receiving antenna in receiving a wave from the direction of the wave normal at the receiver.

Second is polarization coupling. Each raypath that connects the transmitter and receiver is for a single magnetoionic component (ordinary or extraordinary) that has a characteristic elliptical polarization at each point along the path (although the two components are degenerate for the part of the path in free space below or above the ionosphere). Ratcliffe (1959) and Budden (1961, 1985) give formulae for the characteristic polarizations of the two magnetoionic components from the magnetoionic formula [see Chap. II. 3.1.3, Eq. (4.7)]. (For example, the characteristic polarizations are circular for propagation parallel to the Earth's magnetic field, and linear for propagation perpendicular to the Earth's magnetic field.) As the wave leaves the ionosphere on one of these raypaths, it will keep the same polarization, so it will still have the characteristic polarization that was determined at the exit from the ionosphere when it arrives at the receiving antenna. The receiving antenna can receive only a particular polarization from each direction (usually linear, rotated at some angle). Therefore, the antenna receives only part of the signal from the arriving wave, corresponding to one component of the characteristic polarization of the arriving wave. A similar effect occurs at the transmitter, so some signal is lost because of polarization coupling at both the transmitter and the receiver. Polarization coupling coefficients can often be complex, to give a phase shift associated with the coupling.

7 Combining Signals that Propagate Along Separate Raypaths

Once we find all of the raypaths that connect the transmitter and receiver, and calculate the amplitude and phase associated with each one, we can combine them to give the total signal at the receiver. If the signals from the various raypaths are coherent with one another, then we can combine them coherently, allowing the possibility of phase interference. When we combine the signals coherently, the signal for each raypath is represented by a complex number in which the magnitude of the complex number equals the amplitude, and the phase angle of the complex number equals the phase of the signal. Adding the complex numbers gives a single complex number whose magnitude equals the amplitude of the combined signal, and whose phase angle equals the phase of the signal.

If the signals from the various raypaths are really coherent, and if the accuracy of the calculations (including the representation of the medium and the ray theory approximations) is good enough, the resulting calculation, including interference effects, will be an accurate representation of the measured signal strength.

On the other hand, it may not be appropriate to combine the signals coherently, either because the calculations are not accurate enough (especially the phase calculations), or because the multipath signals themselves are inherently not coherent with one another. In practice, it is often difficult to distinguish these two cases, and the criteria for determining whether the signals from the various paths are correlated is beyond the scope of this treatment.

The standard method for combining signals that are incoherent is to add the squares of the amplitudes, ignoring the phases, and to take the square root of the sum as an estimate of the total amplitude, with an undetermined phase. This procedure really gives an estimate of the root-mean-square (rms) average of an ensemble of cases, rather than a quantity that could be realistically compared with a single measurement.

Between these two extreme cases are situations where the signals from various raypaths are partially coherent. Such cases are beyond the scope of the present treatment, but are discussed by Beran and Parrent (1964).

The discussion so far in this section applies mainly to cw (continuous wave) propagation but can be easily extended to pulse propagation. For pulse propagation, the signals from the various raypaths are combined coherently or incoherently, as appropriate, in the same way as for cw propagation, but as a function of time. Pulses that do not overlap in time arrive separately, so there is no possibility for phase interference.

8 Ray Theory

Ray theory is equivalent to the WKB approximation (Budden 1961, 1985; Wait 1962) to the wave equation. Although the method was given its present name after 1926 (Wentzel 1926; Kramers 1926; Brillouin 1926), the method was discovered earlier (Liouville 1836, 1837a,b; Rayleigh 1912; Jeffreys 1923). The method for constructing WKB solutions is sometimes referred to as the phase integral method (Eckersley 1932), the eiconal method (Felsen and Marcuvitz 1973), or the eikonal method (Weinberg 1962).

The following formulation is from Weinberg (1962). In general, we want to find a solution to the system of n coupled linear homogeneous equations:

$$M(q_i, -i\partial/\partial q_i)\psi(q_i) = 0\,, \tag{8.1}$$

where M is a symmetric $n \times n$ matrix, and ψ is an $n \times n$ column vector. The WKB approximation consists in looking for a solution of the form

$$\psi(q_i) = \psi_0(q_i)\exp\left[iS(q_i)\right]\,, \tag{8.2}$$

in which most of the time and spatial dependence of the solution is in the exponential function of S. S is sometimes called the phase integral or the eiconal (or eikonal) function. If we assume, as a first approximation, that all of the space and time dependence is in S, then substituting Eq. (8.2) into Eq. (8.1) gives

$$M(q_i, p_i)\psi_0 = 0\,, \tag{8.3}$$

where

$$p_i = \partial S/\partial q_i\,. \tag{8.4}$$

For Eq. (8.3) to have non-zero solutions requires that

$$H(q_i, p_i) \equiv \det\left[M(q_i, p_i)\right] = 0\,. \tag{8.5}$$

The solution to Eq. (8.4) for S, with the condition (8.5), is given by the phase integral [Eq. (5.2)]. The q_i and the p_i are determined by a ray tracing programme using Hamilton's equations (2.1) with the Hamiltonian (8.5). Hamilton's equations guarantee that if Eq. (8.5) is satisfied at the start of the ray, it will be satisfied thereafter.

The solutions of Eq. (8.3) represent waves that are nearly uncoupled, but are coupled slightly. The WKB approximation arises from substituting Eq. (8.2) into Eq. (8.1), and making an approximation that uncouples these waves. The result (Weinberg 1962) is a transport equation (Felsen and Marcuvitz 1973),

$$\psi_0^T(A_i\partial/\partial q_i + C)\psi_0 = 0\,, \tag{8.6}$$

where ψ_0^T is the transpose of ψ_0,

$$A_i = \partial M/\partial p_i\,, \tag{8.7}$$

$$C = \left(\frac{1}{2}\right)(\partial^2 M/\partial p_i \partial p_j)\,(\partial p_j/\partial q_i)\,, \tag{8.8}$$

repeated indices are summed from 0 to 3, and we have assumed that $M(q_i, p_i)$ is a symmetric matrix.

Equation (8.3) determines the eigenvectors for the WKB solution but not the magnitudes. The usual way to obtain the magnitudes is to let

$$\psi_0(q_i) = f_0(q_i)\,\varphi_0(q_i)\,, \tag{8.9}$$

where $\varphi_0(q_i)$ is a solution of Eq. (8.3), and $f_0(q_i)$ is a scalar function. Substituting Eq. (8.9) into Eq. (8.6) gives (Weinberg 1962)

$$(d/d\tau)\ln f_0 = -(\varphi_0^T \varphi_0)^{-1} \varphi_0^T (A_i \partial/\partial q_i + C)\varphi_0\,, \tag{8.10}$$

where φ_0^T is the transpose of φ_0. (For alternate representations, see Schiff 1955; Budden 1961, 1972, 1985; Budden and Smith 1976; Felsen and Marcuvitz 1973.)

The WKB approximation can be considered the first term in an asymptotic series expansion for the solution to the wave equation. Felsen and Marcuvitz (1973) give the formulae for the asymptotic series.

We can express Eq. (8.1) through Eqs. (8.5) and (8.9) in cartesian coordinates to show the time dependence explicitly:

$$M(t, \boldsymbol{x}, i\partial/\partial\omega, -i\nabla)\psi(t, \boldsymbol{x}) = 0\,, \tag{8.11}$$

$$\psi(t, \boldsymbol{x}) = \psi_0(t, \boldsymbol{x})\exp[iS(t, \boldsymbol{x})]\,, \tag{8.12}$$

$$M(t, \boldsymbol{x}, \omega, \boldsymbol{k})\psi_0 = 0\,, \tag{8.13}$$

$$\omega = -\partial S/\partial t\,, \tag{8.14a}$$

$$\boldsymbol{k} = \nabla S\,, \tag{8.14b}$$

$$H(t, \boldsymbol{x}, \omega, \boldsymbol{k}) \equiv \det[M(t, \boldsymbol{x}, \omega, \boldsymbol{k})] = 0\,, \tag{8.15}$$

$$\psi_0(t, \boldsymbol{x}) = f_0(t, \boldsymbol{x})\varphi_0(t, \boldsymbol{x})\,. \tag{8.16}$$

The corresponding specialization to cartesian coordinates of Eq. (8.6) through Eqs. (8.8) and (8.10) is straightforward, although the notation becomes tedious and is therefore omitted.

The solution [Eq. (8.10)] is equivalent to the focussing calculation in Section 6. The WKB approximation gives the mathematical justification for the focussing calculation.

An alternative method for deriving the WKB approximation comes from the path integral approach for solving the wave equation (Feynman and Hibbs 1965). In the path integral approach, the solution for the field at the receiver is written as an integral over all paths in space–time (not just raypaths) that connect the transmitter and the receiver. Although the path integral method is rigorous, it rarely gives a practical method for solving the wave equation. Its main use is in deriving approximate solutions such as the WKB approximation.

The idea of the path integral formulation is that all paths that connect the transmitter and receiver contribute to the total field, but some paths contribute much more than others. If we make the saddlepoint approximation (Budden 1961, 1985; Felsen and Marcuvitz 1973; Wait 1962) to evaluate the path integral (not really a single integral, but an infinite number of integrals), then we restrict consideration to only those paths that contribute the most to the signal at the receiver. In this case, the saddlepoint approximation chooses those paths for which the action [proportional to the phase integral, Eq. (5.2)] is stationary for infinitesimal variations of the path, keeping the endpoints fixed. This is equivalent to Hamilton's principle of least action, which is called Fermat's principle when applied to wave propagation. These paths are found by using Hamilton's equations. See Feynman and Hibbs (1965) for details. The results are the same as those given above.

If we start with a system of equations such as Eq. (8.1), and make a change of variable (either to a dependent or to an independent variable), we will obtain a different set of equations that are equivalent to the original. Although the solutions to the new set of equations will be different (because the variables are different), using the variable conversion to find the original set of variables from the solutions for the new variables gives the same solutions that would have been obtained from the original set of equations, so there is no contradiction.

The difficulty is that the transformed equations will give a different Hamiltonian, a different raypath, and a different WKB approximation, *even for the same variables* than were obtained from the original equations. The two WKB approximations agree within the error of the WKB approximation, however. Pierce (1965), Einaudi and Hines (1970), Budden and Smith (1976) and Jones (1983) discuss this interesting phenomenon.

For propagation in dissipative media, the wave equation (8.1), the dispersion relation and the Hamiltonian will be complex. Hamilton's equations then lead to raypaths that have complex coordinates. Through analytic continuation into the complex plane for all of the variables involved, all of the calculations including the WKB approximation will also be valid (Budden and Jull 1964). Jones (1970) and Budden and Terry (1971) show applications of these kinds of calculations using ray tracing in complex space. Usually, ray tracing in complex space is needed whenever the gradient in conductivity is significant in refracting the wave such as for the propagation of LF (30–300 kHz) radio waves in the *D* region of the ionosphere (Jones 1970). See also Felsen and Marcuvitz (1973), Suchy (1972a,b) and White and Pedersen (1981) for discussions of ray tracing in complex space. Propagation in a dissipative medium can cause a frequency shift of a pulse, even for a time-independent medium (Jones 1981).

Whenever the gradient in conductivity has negligible effect in refracting the wave (e.g. only the gradient in the dielectric constant is significant in refracting the wave), ray tracing in complex space is not required. In that case, it is sufficient to define a real Hamiltonian that approximates the complex dispersion relation and to use ordinary ray tracing to calculate the WKB approximation, even for propagation in dissipative media.

There are well-known limitations to ray theory, that is, situations where the WKB approximation is not valid. The most common is at a caustic, where ray theory predicts an infinite signal strength. (This is discussed in Sect. 6).

There is a general criterion for the validity of the WKB approximation in terms of Fresnel zones. Each raypath can be considered to be surrounded by a region called the first Fresnel zone, which consists of all paths connecting the transmitter and receiver, whose phase differs from that along the raypath by less than 180°. Although, strictly speaking, the Fresnel zone is not a region of space, but a set of paths, we may often refer to the region of space containing those paths as the Fresnel zone.

In general, the WKB approximation breaks down if (1) the medium changes too quickly within the first Fresnel zone, or (2) the first Fresnel zones from two raypaths overlap.

In the first case, diffraction is significant; in the second, the receiver is too close to a caustic. In an alternative terminology, the receiver is said to be in the Airy region of the caustic in the second case (Budden 1961, 1985). In still different terminology, both of the above cases result when the saddlepoint in the path integral is not an isolated saddlepoint (Budden 1961, 1985; Felsen and Marcuvitz 1973).

9 A Ray Tracing Programme as an Example

The three-dimensional ray tracing programme for radio waves in the ionosphere (Jones and Stephenson 1975) gives an example of a ray tracing programme. This programme has the following properties:

- It integrates Hamilton's equations in four dimensions (including time for a time-dependent medium), but ignores the effect of the frequency shift on the propagation, and assumes that the time rate of change of the medium does not change during the time of propagation from the transmitter to the receiver.
- It uses Hamilton's equations in Earth-centered spherical polar coordinates.
- Equations (3.8) and (3.9) give the Hamiltonian used. However, there is an option to switch to a different Hamiltonian near a Spitze (Poeverlein 1949, 1950; Davies 1965, 1969).
- It provides six choices for the refractive index formula:

  1. Magnetoionic formula for constant collision frequency, including the effects of the Earth's magnetic field and collisions between electrons and neutral molecules.
  2. Magnetoionic formula, neglecting collisions.
  3. Magnetoionic formula for constant collision frequency, neglecting the Earth's magnetic field.

4. Magnetoionic formula, neglecting both the Earth's magnetic field and collisions.
5. Sen–Wyller formula, including the Earth's magnetic field and collisions.
6. Sen-Wyller formula, including collisions but neglecting the Earth's magnetic field.

- In addition to the raypath (including pulse travel time), the programme calculates, optionally, phase, dissipation, frequency shift due to a time-dependent medium, and geometrical path length.
- The approximation (5.9) is used to calculate dissipation.
- Models of electron density, the Earth's magnetic field, and collision frequency are expressed in Earth-centered spherical polar coordinates (essentially in terms of height, longitude and latitude). Thus, refractive index is specified relative to the Earth, not relative to the transmitter location.
- Several of each of the three kinds of models are furnished with the programme, but the user can add new models by writing new subroutines.
- The transmitter can be located at any height, longitude and latitude.
- The radio frequency, azimuth launch angle, and elevation launch angle can be swept in ranges and increments specified by the user.
- The receiver can be specified at any height.
- Computer-readable output is optionally generated at each ground reflection and each time the ray crosses the receiver height. This output is read by separate programs that use interpolation to find eigenrays that connect the transmitter with a specified receiver.
- A detailed printout of the raypath is optionally available.
- Plots of the raypath are available, showing the projection of the raypath on any vertical or horizontal plane specified.
- The numerical integration algorithm uses an Adams–Moulton predictor–corrector method with a Runge–Kutta starter. The user can specify the maximum allowable single-step error.
- A sample case is included to illustrate and test the programme on the user's computer.
- The programme is written in FORTRAN, and is easy to alter for specific needs because of its modular organization into subroutines.

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II.3.2 Radio Frequencies

II.3.2.1 Observations of the Middle Atmosphere and Lower Thermosphere by Radars

1 Introduction

Radars operating in the frequency ranges from MF (medium frequency) to UHF (ultra-high frequency) are used to investigate the structure and dynamics of the troposphere, stratosphere, mesosphere and thermosphere. In Table 1 the different

Table 1. Radar methods for investigations of the middle atmosphere and the lower thermosphere

	Typical operation parameters (approximate)				
Method	Frequency range	Wavelength λ [m]	Average power [kW]	Antenna dimension λ	Height region
MF radar	MF–HF	150–50	0.01– 1.0	2– 10	M, LT
Meteor radar	HF–VHF	10– 6	0.1– 10	2– 10	M, LT
MST radar	VHF	6– 7	1–100	5– 50	M, S, T
ST radar	UHF–SHF	0.7– 0.10	50–500	10–500	S, T
Incoherent scatter radar	VHF–UHF	1.4– 0.25	100–300	100–300	M, LT

MF = 0.3–3.0 MHz
HF = 3.0–30 MHz
VHF = 30–300 MHz
UHF = 30–3000 MHz
SHF = 3–30 GHz

M = Mesosphere
S = Stratosphere
T = Troposphere
LT = Lower Thermosphere